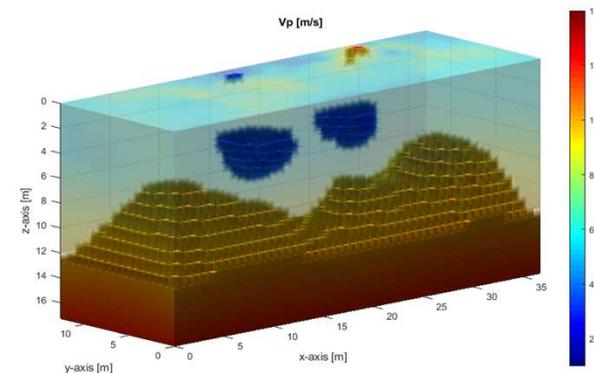
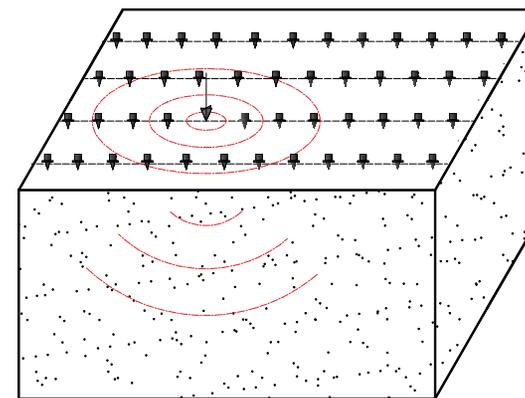


# 3D full-waveform inversion of Thumper source wavefields

Workshop on Shared-Use of  
NHERI@UTexas Mobile Shakers  
for Geophysical and  
Seismological Research

December, 3<sup>rd</sup>, 2030

by  
Khiem Tran, Ph.D.  
University of Florida





# Outline of presentation

- Need for site investigation
- FWI motivation
- FWI challenges at geotechnical scales
- Overview of 3D FWI
- 3D FWI in hybrid time-frequency domain
- Field data application with Thumper source
- Conclusion

# Need of site investigation

## Problem

- Structural collapses that lead to significant property damage and even fatalities

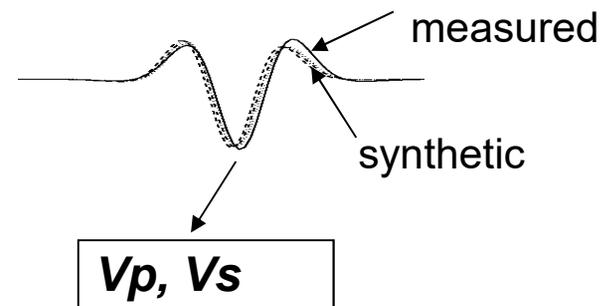
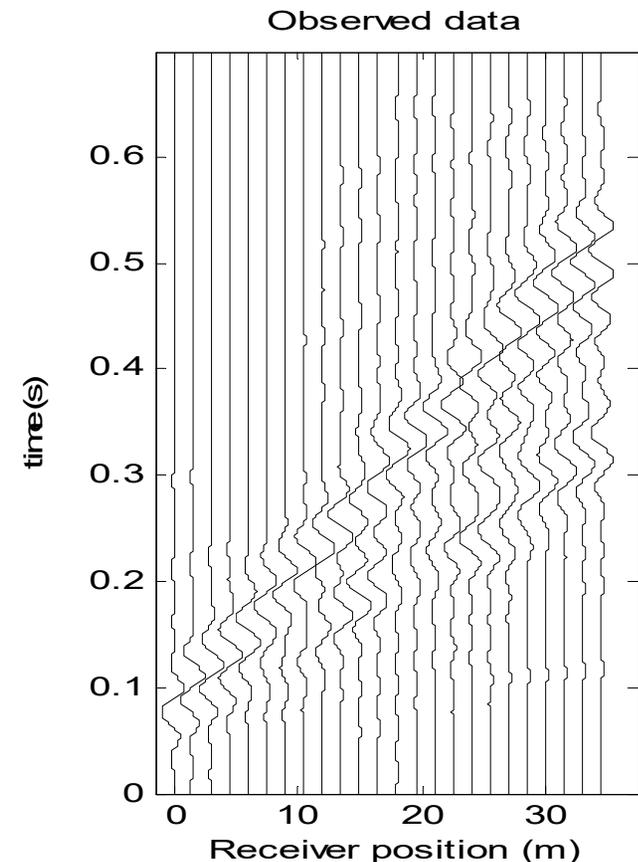
## Site investigation

- Typical invasive testing SPT, CPT – tests  $< 0.1\%$  of material
- Seismic methods can test over large volume of materials
- Soil/rock property and stratigraphy, and embedded voids/anomalies



# FWI Motivation

- Conventional seismic methods analyse travel times of certain wave types
  - inversion of P-wave first arrival travel time
  - inversion of surface wave dispersion
  - migration
  - use only phase, not magnitude
- FWI is wave-equation based and has the potential to
  - use full information content (waveforms), both phase and magnitude
  - consider all measured wave types (P-, S-, Rayleigh waves)
  - **characterize both  $V_p$  and  $V_s$  at high resolution (meter pixel)**



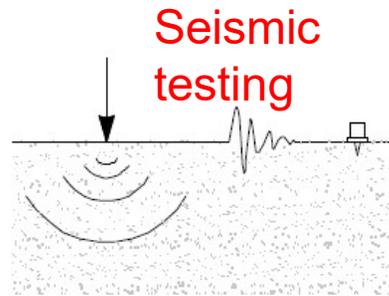


## **FWI challenges at geotechnical scales**

- inconsistent wave excitation, unknown source signatures (inversion artifacts near source locations)
- strong variability of near surface soil/rock, poor priori information in the initial model (shallow inversion artifacts, local minimum)
- dominant Rayleigh waves, small body waves with strong attenuation (large model updates at shallow depths, poorly resolved deeper structures)
- Need strong, broadband, consistent seismic source (e.g. mobile shakers)

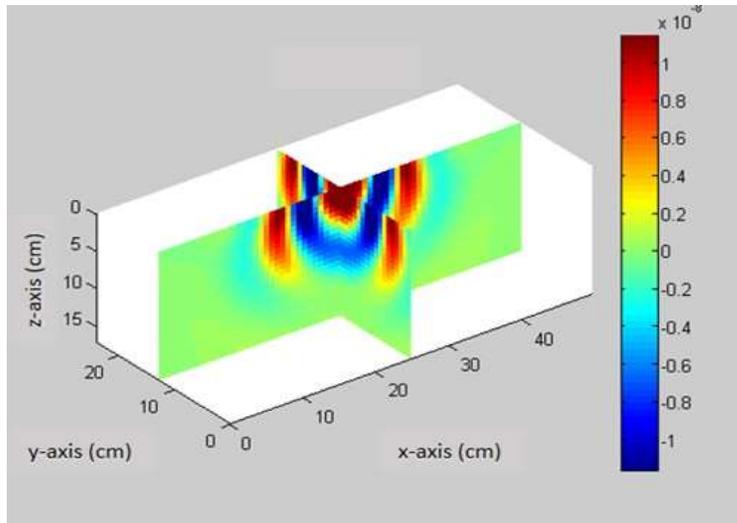
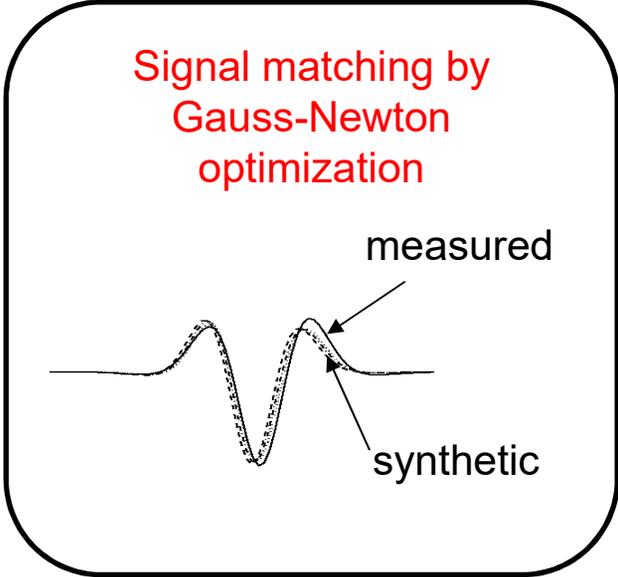
# 3D FWI Method

Material properties  
 $V_s$ ,  $V_p$   
?

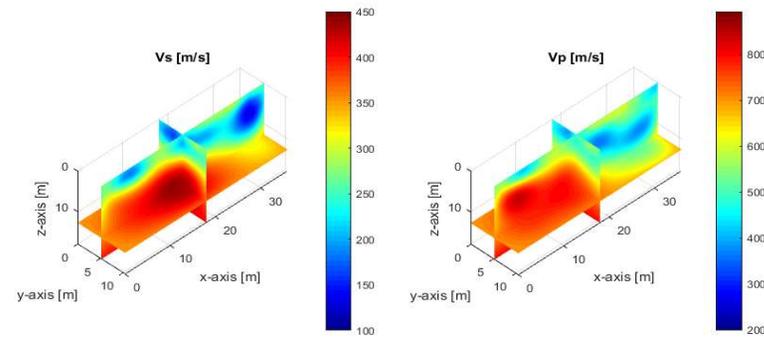


Measured data

Synthetic data



3D wave propagation



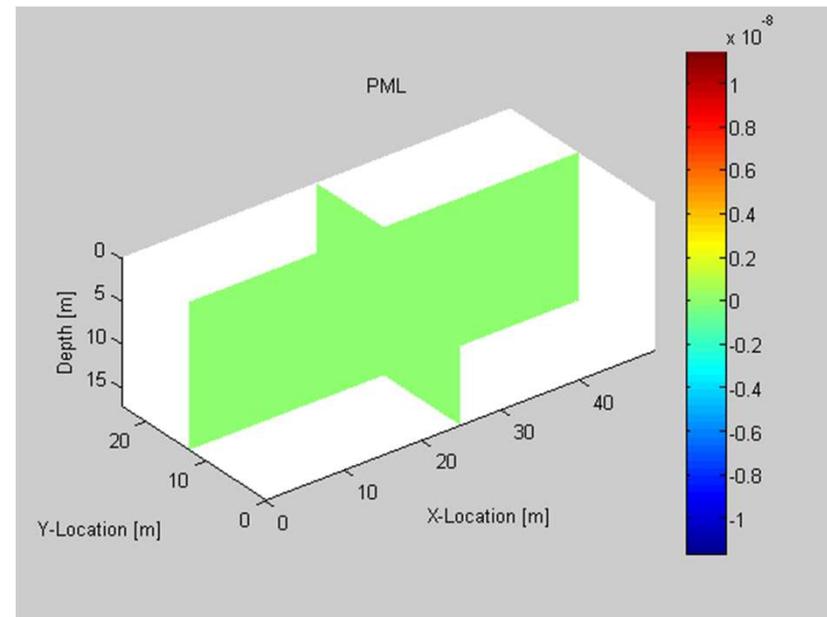
# 3D FWI method

- Forward modeling in **time-domain**

$$\rho \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i \quad \text{where } i, j = 1, 2, 3$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \lambda \frac{\partial v_k}{\partial x_k} + 2\mu \frac{\partial v_i}{\partial x_j} \quad \text{if } i \equiv j$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad \text{if } i \neq j$$



PML is used at bottom and 4 vertical boundaries.

# 3D FWI method

## ➤ Model updating by Gauss-Newton in **frequency-domain**

- Velocity residual:  $\Delta \tilde{\mathbf{d}}_{s,r} = \tilde{\mathbf{F}}_{s,r}(\mathbf{m}) - \tilde{\mathbf{d}}_{s,r}$

- Misfit function: 
$$E(\mathbf{m}) = \frac{1}{2} \Delta \tilde{\mathbf{d}}^t \Delta \tilde{\mathbf{d}}$$
 Filter, focus, balance gradient vector, as a weighting function

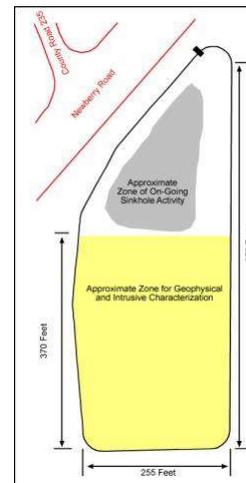
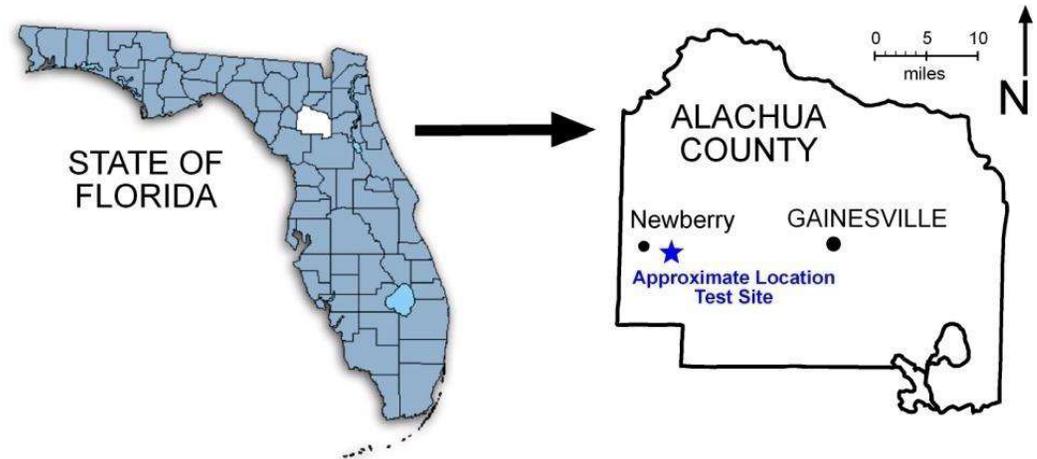
- Model updating: 
$$\mathbf{m}^{n+1} = \mathbf{m}^n - \alpha^n \left[ \tilde{\mathbf{J}}^t \tilde{\mathbf{J}} + \lambda_1 \mathbf{P}^t \mathbf{P} + \lambda_2 \mathbf{I}^t \mathbf{I} \right]^{-1} \tilde{\mathbf{J}}^t \Delta \tilde{\mathbf{d}}$$

- Jacobian matrix  $\tilde{\mathbf{J}}$  is the frequency-domain partial derivative wavefield with respect to model parameter  $\mathbf{m}$  ( $V_s$ ,  $V_p$  of cells)

Tran K.T., Nguyen D.T., Hiltunen D.R., Stokoe K., and Menq F. (2020) "3D full-waveform inversion in time-frequency domain: field data application", *Journal of Applied Geophysics*, Vol. 178, 104078

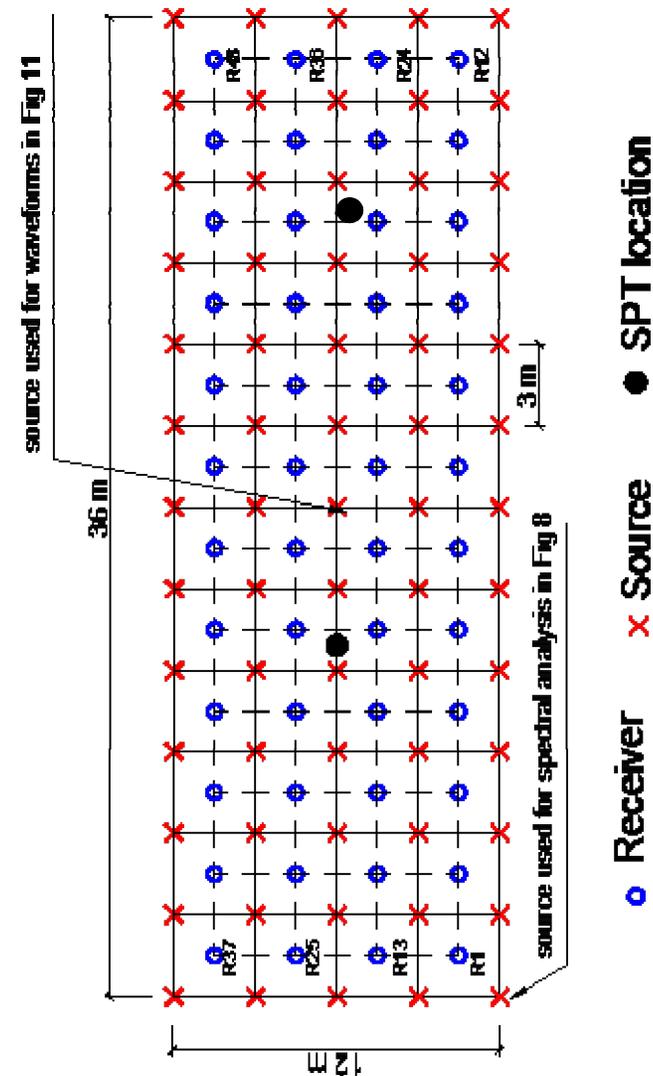
# Newberry site

- Dry retention pond in Newberry, FL
- Mix sand and clay over lime stone bedrock
- Site was marked by 25 lines (A to Y) at 3 m spacing
- Data were collected by NHERI @UTexas team using 48 4.5 Hz vertical geophones and Thumper source.

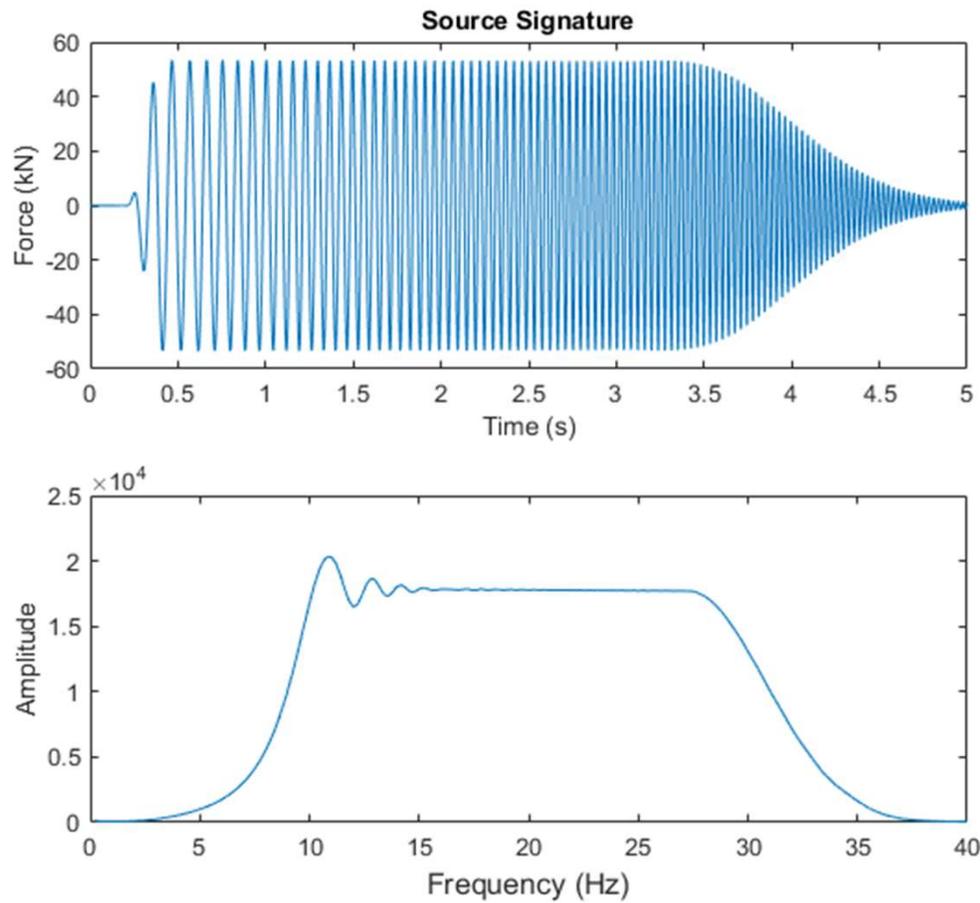


# Seismic Survey at Lines O to S

- Test area of 36 x 12 m
- 48 geophones located in 12 x 4 grid
- 65 shots located in 13 x 5 grid
- Thumper source at 8 to 80 Hz

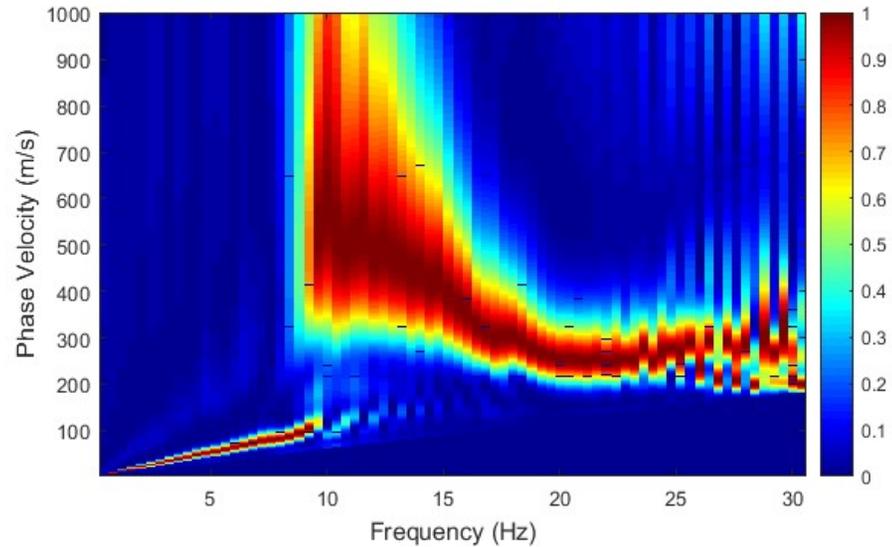


# Thumper source

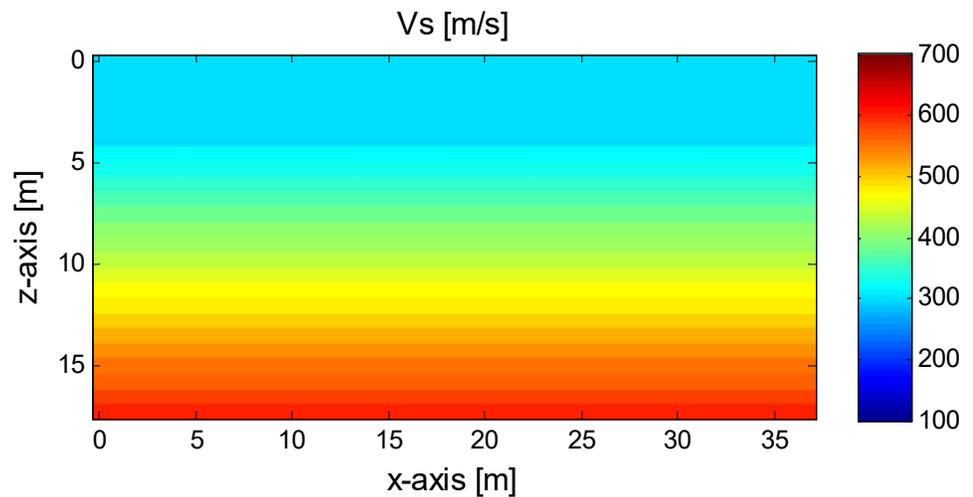


# Newberry data analysis

- 2 inversion runs  
The first run at 12, 15, 18 Hz  
The second run at 20, 25, 30 Hz
- 32 hours on a desktop computer  
(32 cores of 3.46 GHz each  
and 256 GB of memory)

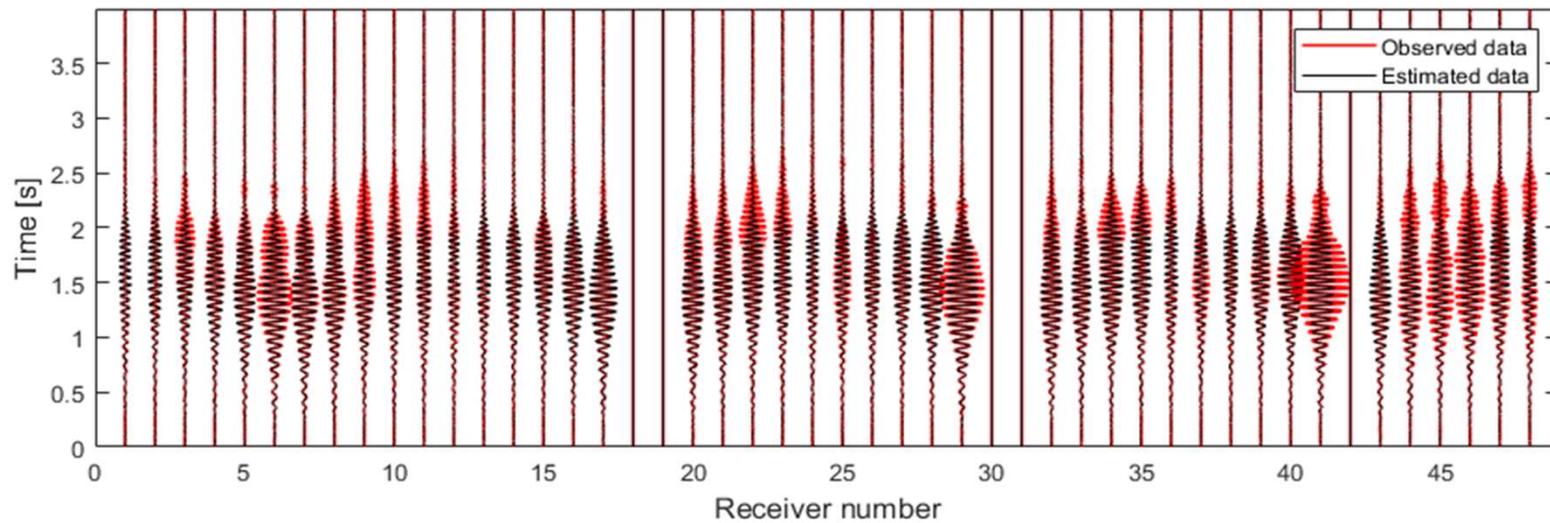
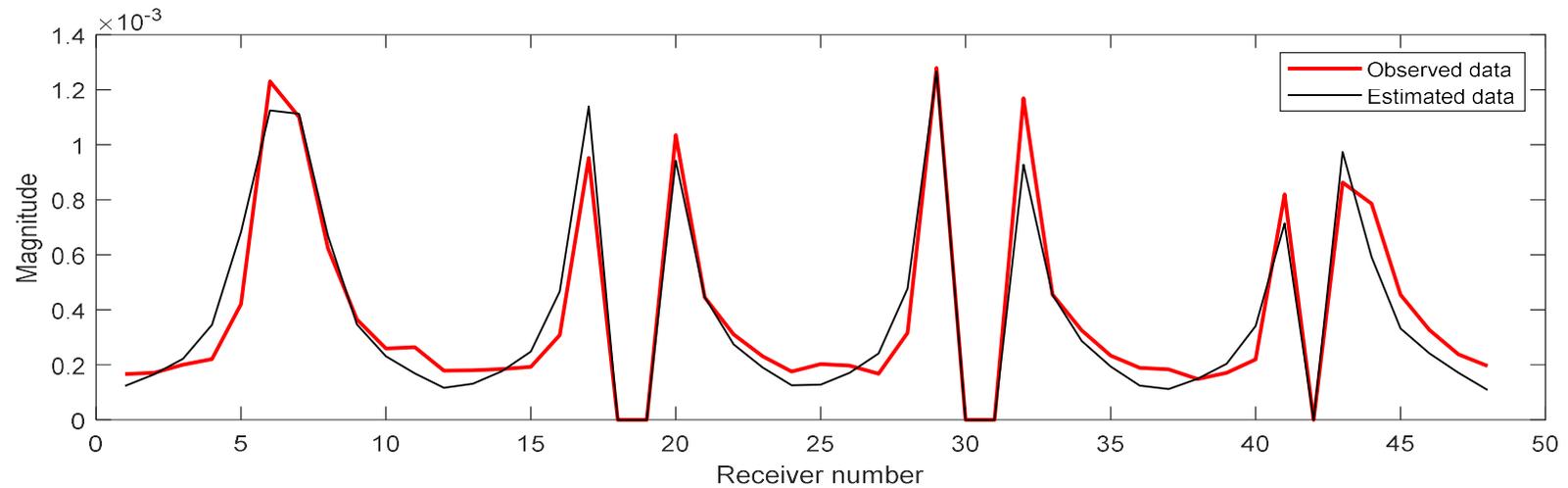


■ Power spectrum



■ Initial model

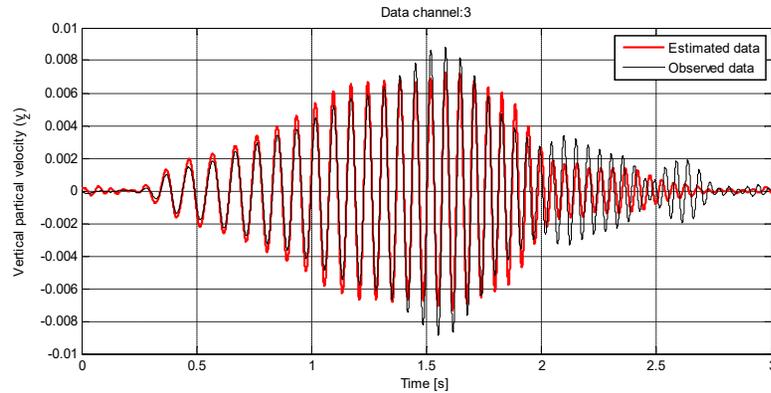
# Newberry: data analysis



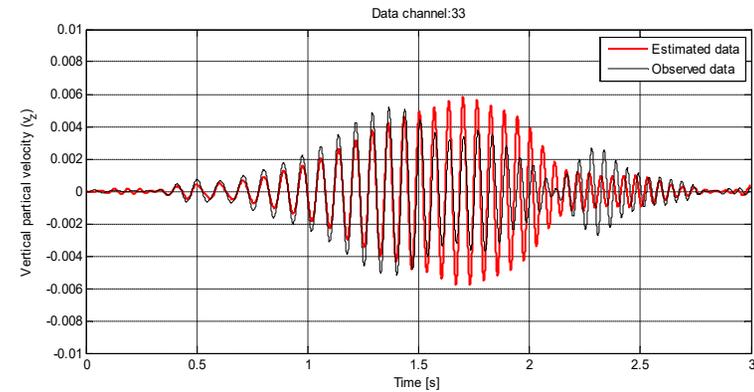
Waveform comparison for the middle shot

# Newberry: data analysis

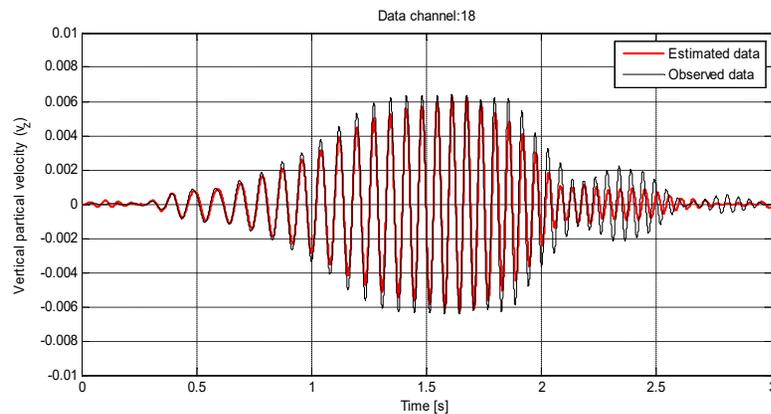
Channel 3, Line OP, x= 7.5m, y= 1.5m



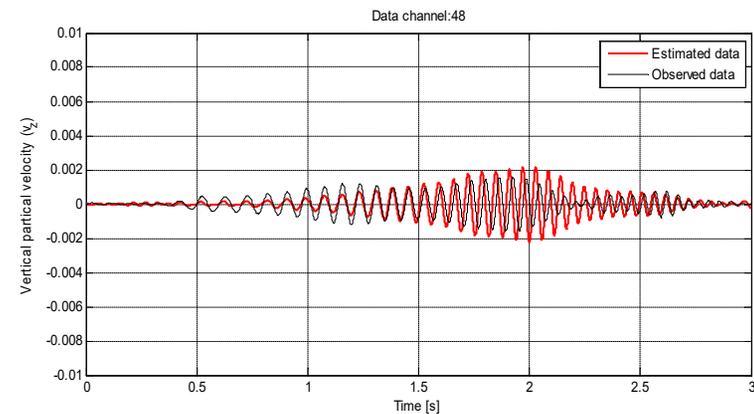
Channel 33, Line QR, x= 25.5m, y= 7.5m



Channel 18, Line PQ, x= 16.5m, y= 4.5m

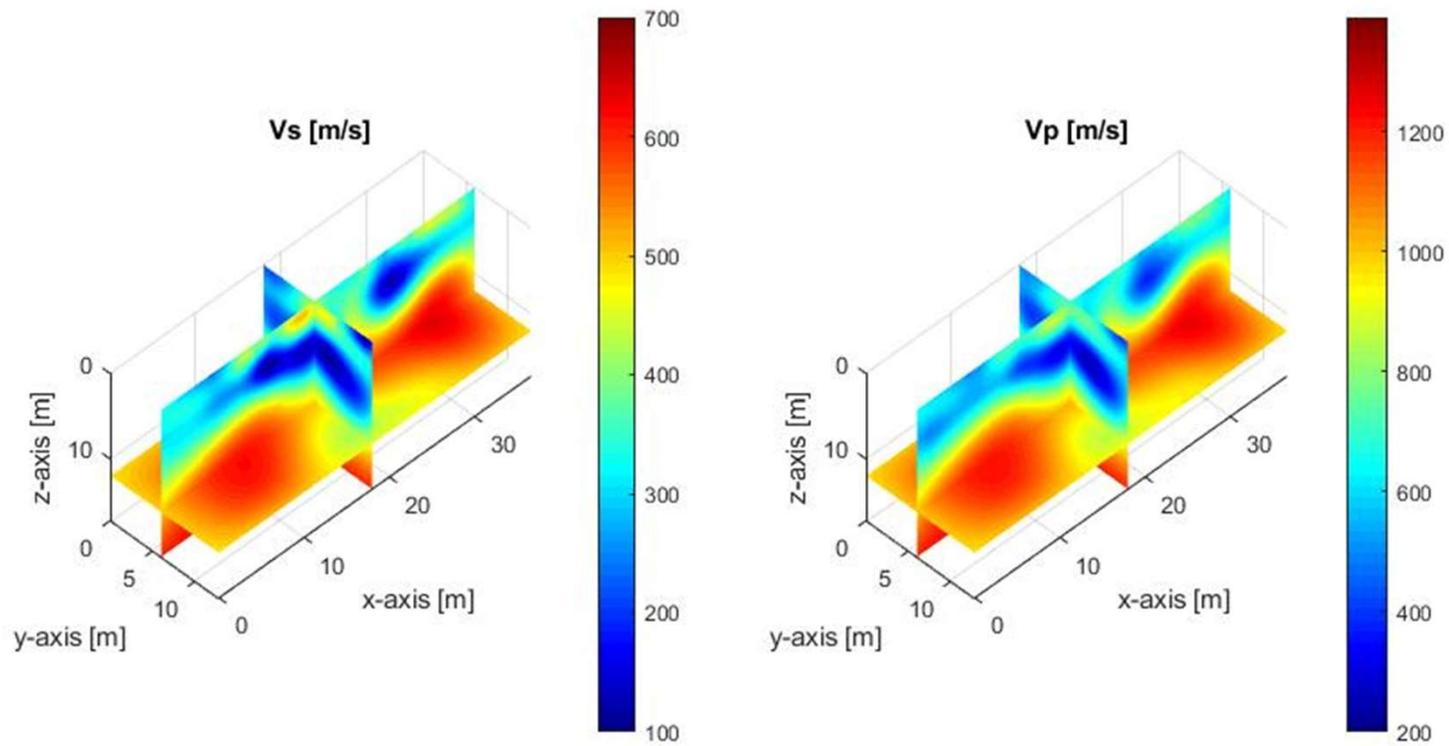


Channel 48, Line RS, x= 34.5m, y= 10.5m

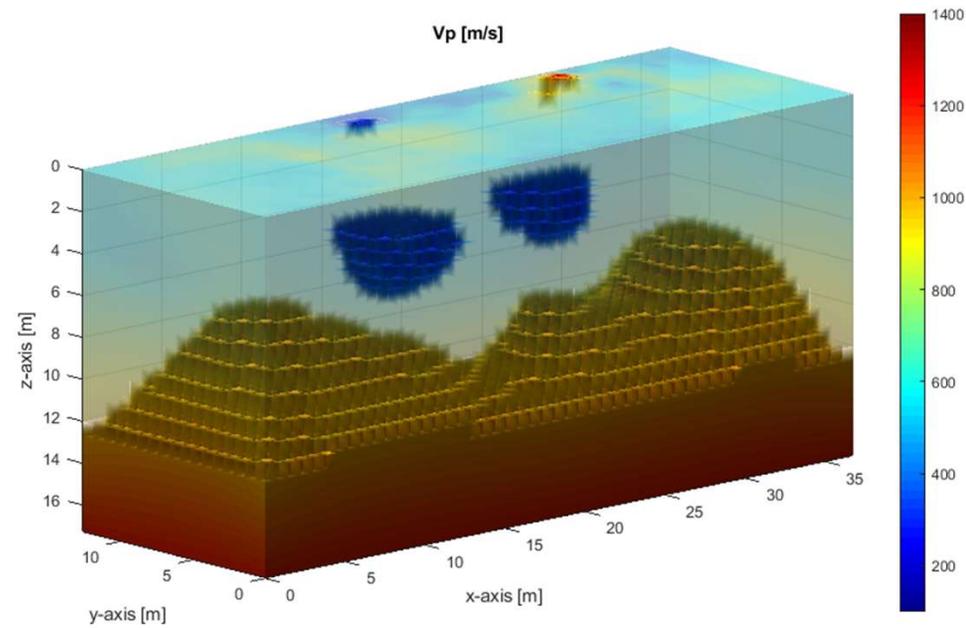
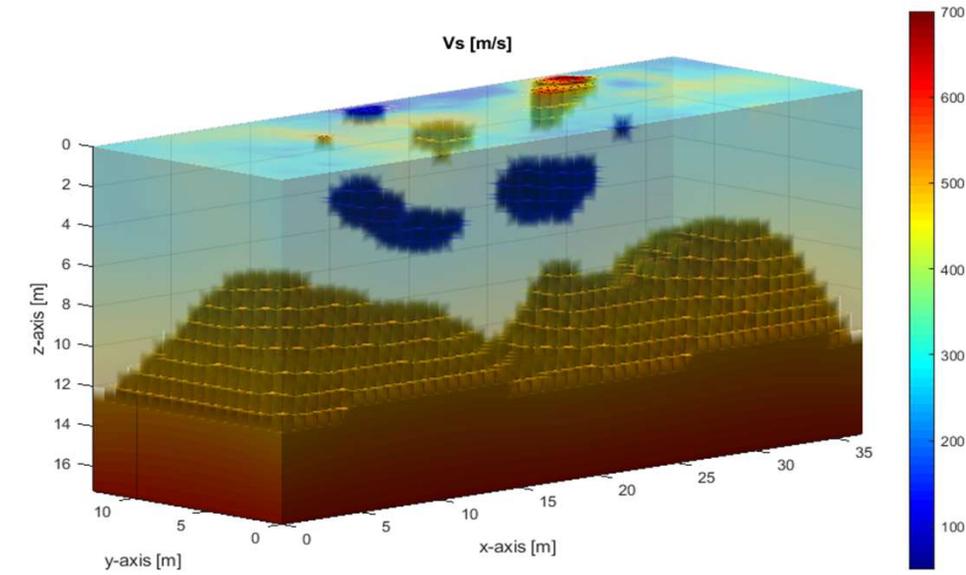


Waveform comparison for 4 sample channels for shot 1 at x=0m, y=0m

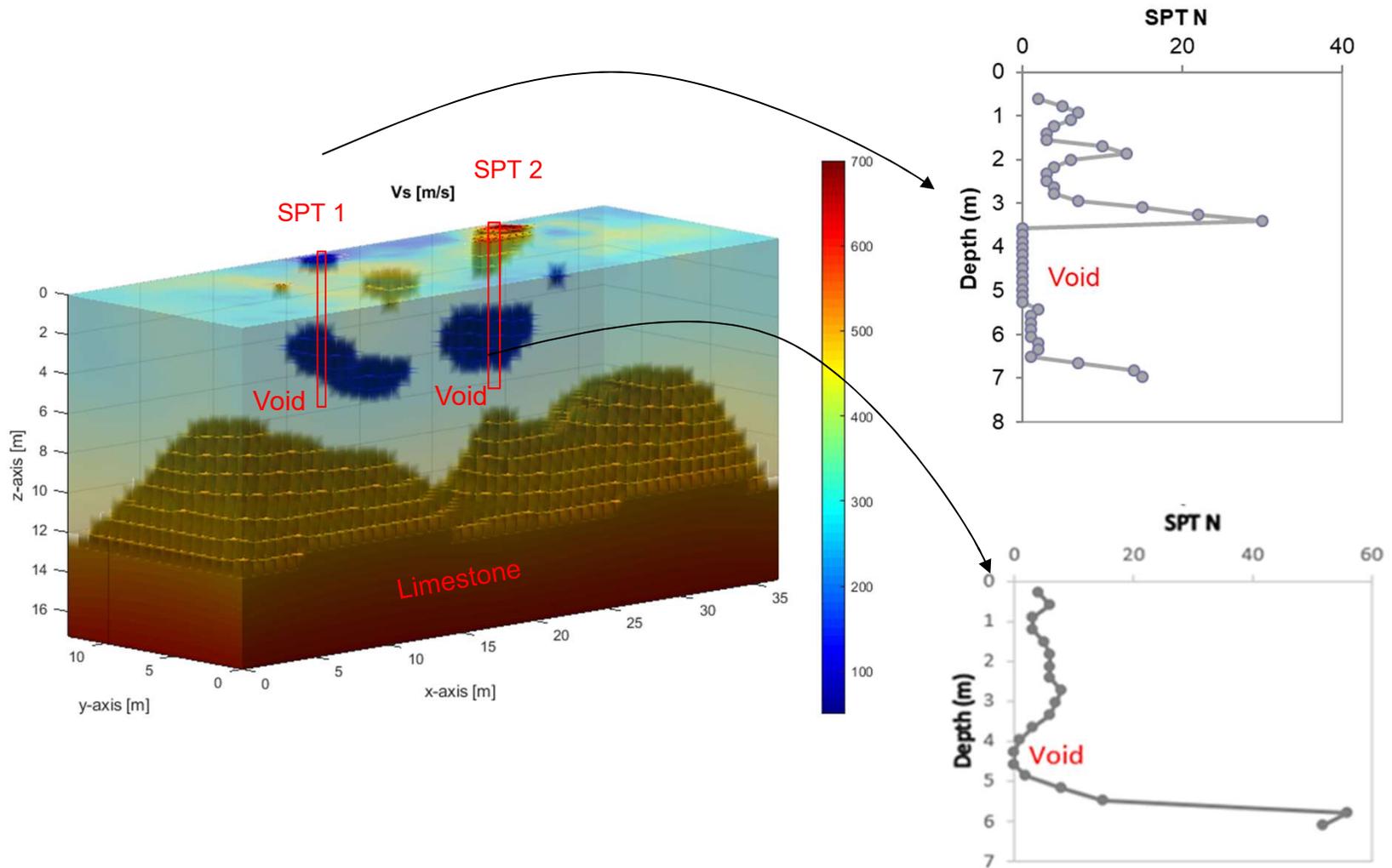
# Newberry: 3D FWI Results



# Newberry: 3D FWI Results



# SPT comparison





## Conclusion

- Thumper source produces strong, broadband, consistent wavefields required for time-frequency 3D FWI
- Both  $V_s$  and  $V_p$  can be characterized at high resolution (meter pixel) to 18 m in depth
- The field seismic results well agree with invasive SPT N-values, including the depth of bedrock and identification of buried voids



# Acknowledgments

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- PhD students Trung Nguyen, Majid Mirzanejad

# *Thank You!*

