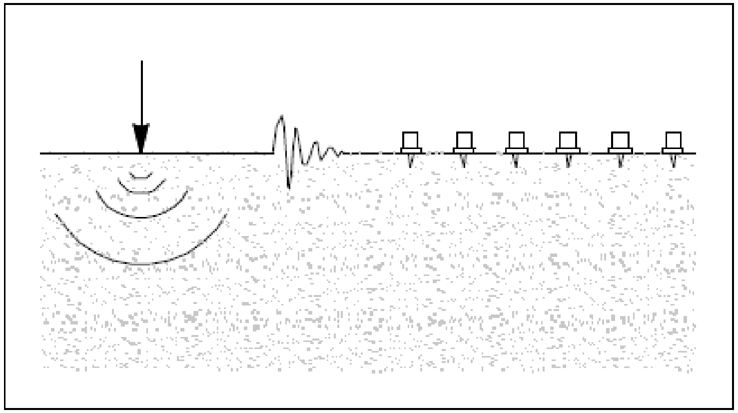
Site Characterization with 3D Full Seismic Waveform Tomography

Stiffness-based ground deformation predictions workshop
NHERI@UTexas
Seattle 2018



by

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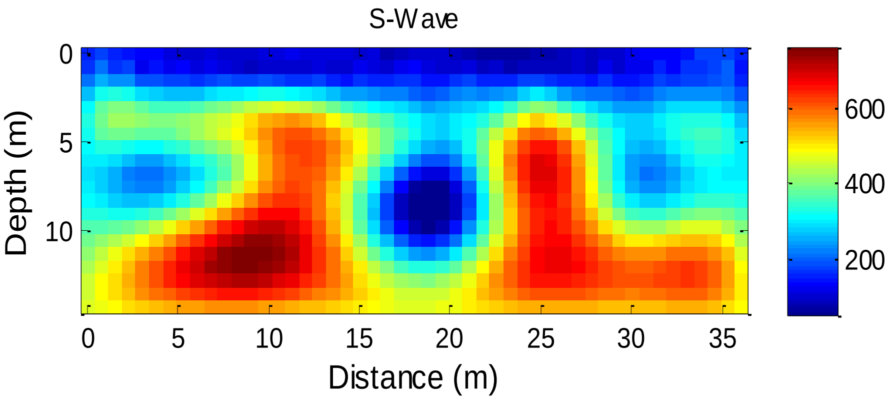


Outline of presentation

- Need for site investigation
- FWI motivation
- FWI challenges at geotechnical scales
- Overview of FWI
- 3D waveform tomography methods
    - 3D FWI using Adjoint gradient
    - 3D FWI using Gauss-Newton
    - Synthetic data applications
    - Field data applications
- Conclusion

Need of site investigation

Problem

- Structural collapses that lead to significant property damage and even fatalities

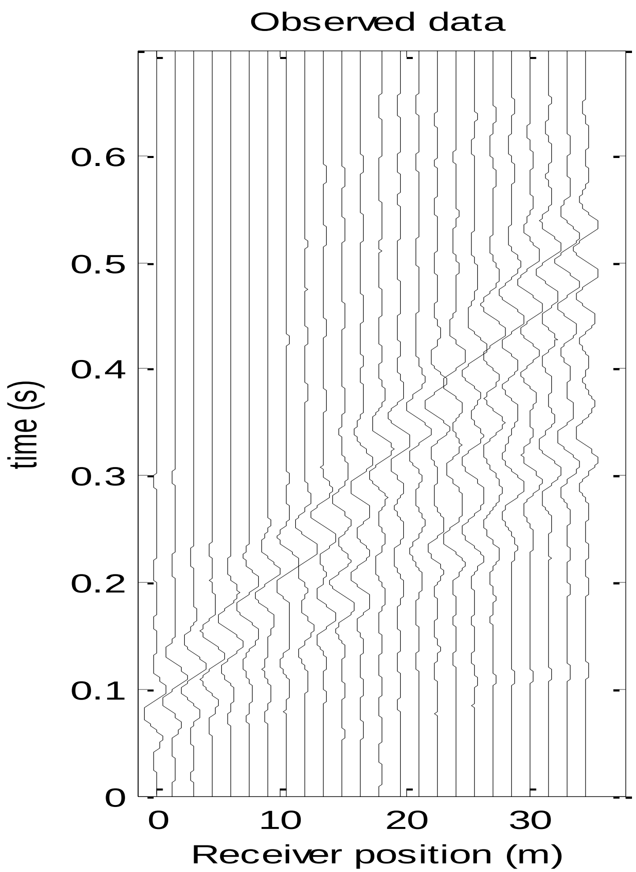
Site investigation

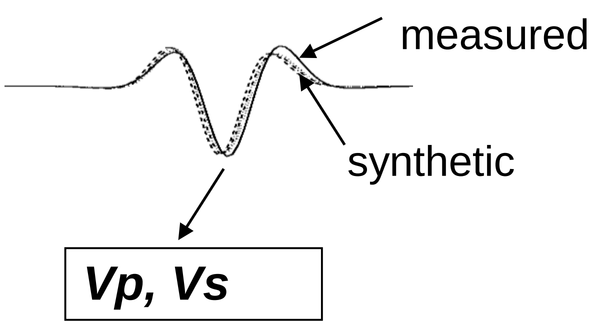
- Typical invasive testing SPT, CPT – tests < .1% of material
- Seismic methods can test over large volume of materials
- Soil/rock property and stratigraphy, and embedded voids/anomalies

Sinkhole collapses

FWI Motivation

- Conventional seismic methods analyse travel times of certain wave types
  - inversion of P-wave first arrival travel time
  - inversion of surface wave dispersion
  - migration
  - use only phase, not magnitude
- FWI is <u>wave-equation based</u> and has the potential to
  - use full information content (waveforms), both phase and magnitude
  - consider all measured wave types (P-, S-, Rayleigh waves)
  - **characterize both Vp and Vs at high resolution (meter pixel)**





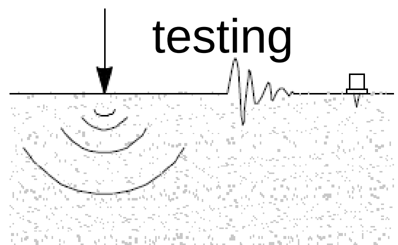
FWI challenges at geotechnical scales

- inconsistent wave excitation, unknown source signatures (inversion artifacts near source locations)
- strong variability of near surface soil/rock, poor priori information in the initial model (shallow inversion artifacts, local minimum)
- dominant Rayleigh waves, small body waves with strong attenuation (large model updates at shallow depths, poorly resolved deeper structures)

Overview of full waveform inversion

Vs, Vp
(model **m**)
?

Seismic
testing

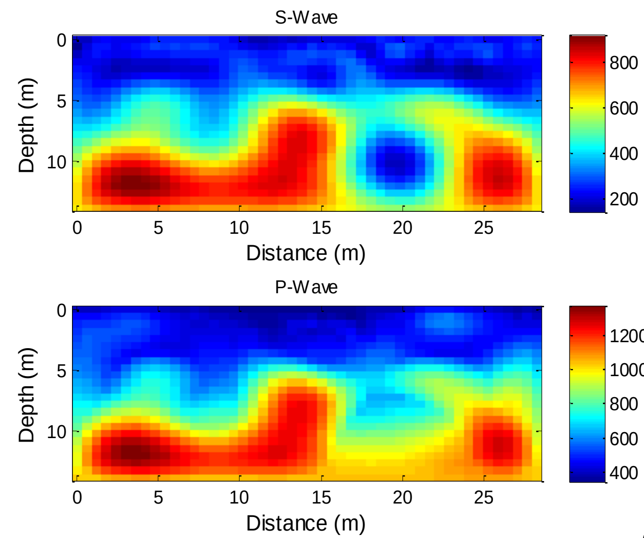


Measured
wave field
d

Inverse problem

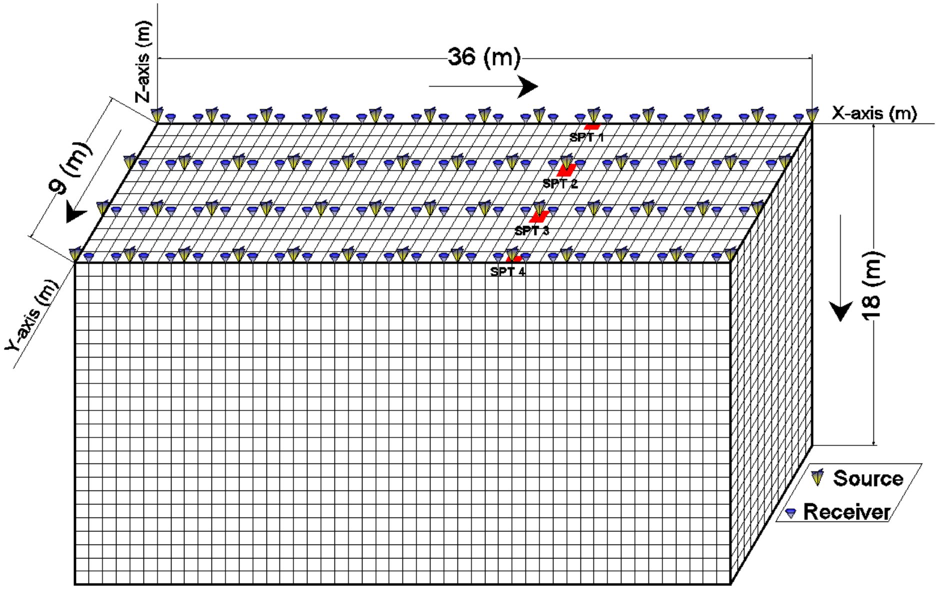
Inversion method:

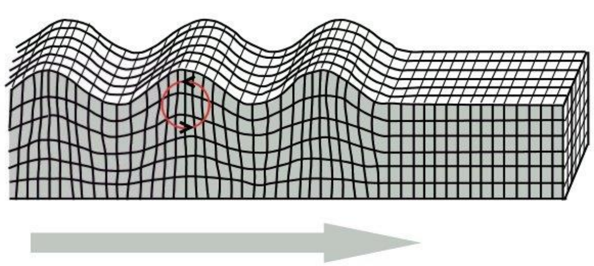
- ➢ Forward modeling **d** = f(**m**)
  - 3D elastic wave equations
    $\longrightarrow \mathbf{d}_{est} = f(\mathbf{m}_{est})$
- ➢ Model updating to match $d_{est} \approx \mathbf{d}$
  - Global optimization: simulated annealing, genetic algorithm, ANN
  - Deterministic optimization: Gradient, Newton, Gauss-Newton methods

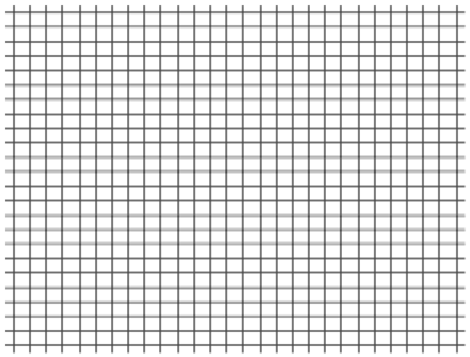


Data Acquisition

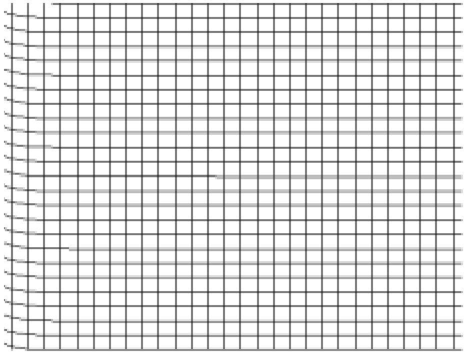
- 2D grids of sources & receivers at 1 to 3 m spacing
- Propelled energy generator or Shaker (5-80 Hz signals)
- P-, S-, and Rayleigh waves are all recorded and used for analysis







P-wave

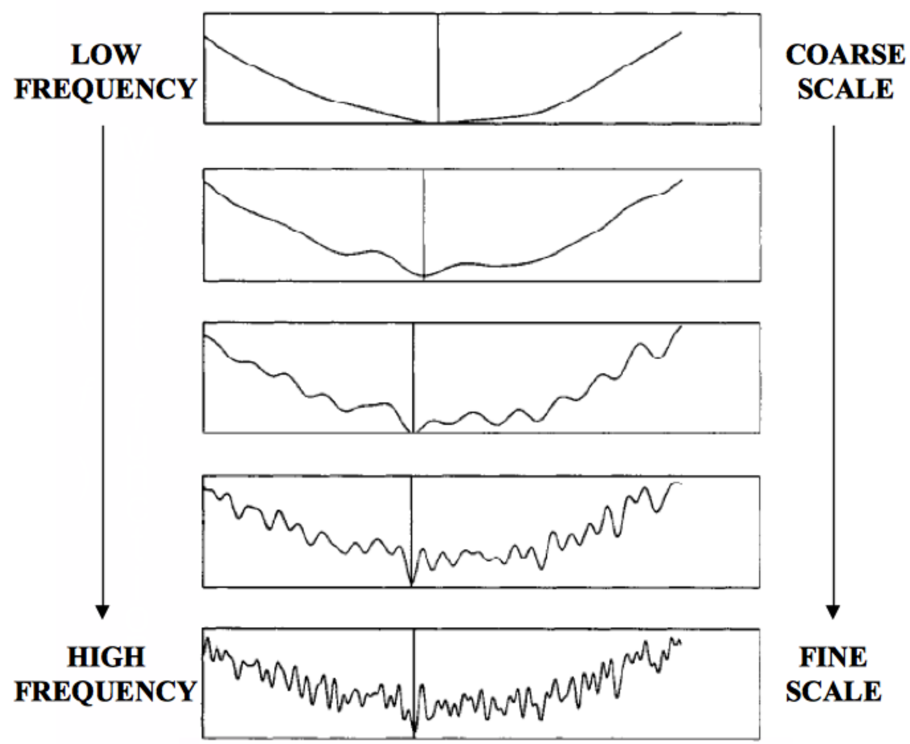


S-wave

Data Analysis

- Start analysis at lowest frequencies and move up
- Low frequencies (large wavelengths) require less detailed information of initial model
- Adding high frequency data gradually helps to resolve detailed features and improve resolution

Misfit function



Bunks et al. (1995)

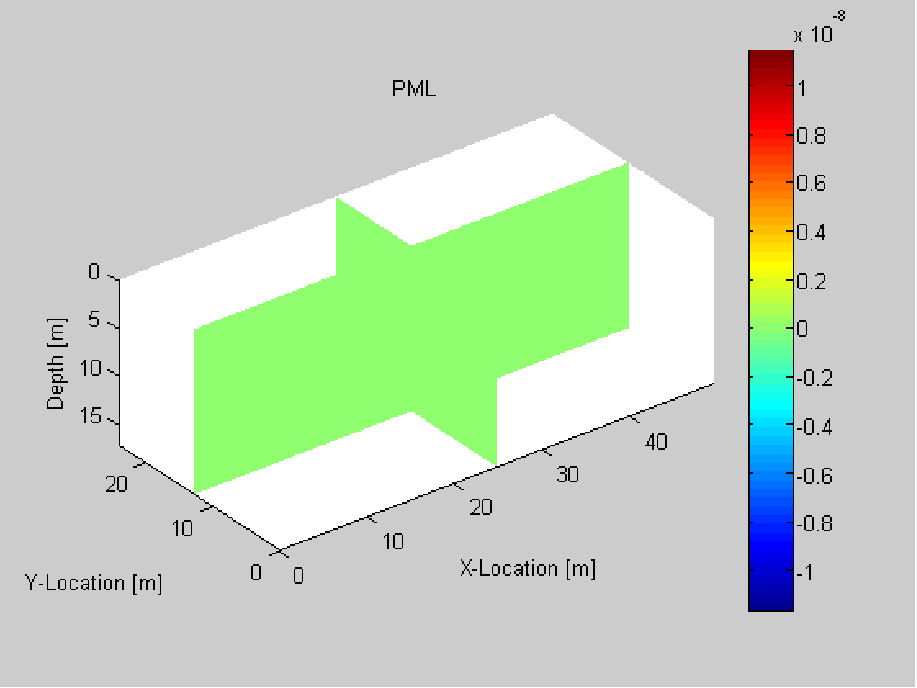
3D FWI

➢ Forward modeling
by 3D wave equations

$$\rho \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i \quad where\ \ i, j = 1,2,3$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \lambda \frac{\partial v_k}{\partial x_k} + 2\mu \frac{\partial v_i}{\partial x_j} \qquad \text{if} \quad i \equiv j$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \qquad \text{if} \quad i \neq j$$



Perfectly Matched Later (PML) is used
at bottom and 4 vertical boundaries.

Nguyen D.T. and Tran K.T. (2018), “Site Characterization with 3-D Elastic Full Waveform Tomography”, *Geophysics*, Vol. 83 (5), pp. R389–R400

3D FWI

- Model updating by Adjoint Gradient

Displacement residual:
$$\Delta u_{i,j}(t) = \int_0^t F_{i,j}(\boldsymbol{m},\tau)d\tau - \int_0^t d_{i,j}(\tau)d\tau$$

Misfit function:
$$E(\boldsymbol{m}) = \frac{1}{2}\Delta\boldsymbol{u}^t\Delta\boldsymbol{u}, \text{ where } \Delta\boldsymbol{u} = \{\Delta u_{i,j}, i = 1,..,NS,\ \ j = 1,..,NR\}$$

Gradients for Lame's parameters:

$$\delta\lambda = -\sum_{i=1}^{NS}\int_0^T dt\left[\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)\left(\frac{\partial \psi_x}{\partial x}+\frac{\partial \psi_y}{\partial y}\right)+\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}\right)\left(\frac{\partial \psi_x}{\partial x}+\frac{\partial \psi_z}{\partial z}\right)+\left(\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)\left(\frac{\partial \psi_y}{\partial y}+\frac{\partial \psi_z}{\partial z}\right)\right]$$

$$\delta\mu = -\sum_{i=1}^{NS}\int_0^T dt\left[\begin{array}{c}\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)\left(\frac{\partial \psi_x}{\partial y}+\frac{\partial \psi_y}{\partial x}\right)+\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right)\left(\frac{\partial \psi_x}{\partial z}+\frac{\partial \psi_z}{\partial x}\right)+\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right)\left(\frac{\partial \psi_y}{\partial z}+\frac{\partial \psi_x}{\partial z}\right)\\ +2\left(\frac{\partial u_x}{\partial x}\frac{\partial \psi_x}{\partial x}+\frac{\partial u_y}{\partial y}\frac{\partial \psi_y}{\partial y}+\frac{\partial u_z}{\partial z}\frac{\partial \psi_z}{\partial z}\right)\end{array}\right]$$

Nguyen D.T. and Tran K.T. (2018), "Site Characterization with 3-D Elastic Full Waveform Tomography", *Geophysics*, Vol. 83 (5), pp. R389–R400

3D FWI

➢ Model updating by Adjoint Gradient

Gradients for Vs, Vp: $\delta V_P = 2\rho V_P \delta\lambda$

$$\delta V_S = -4\rho V_S \delta\lambda + 2\rho V_S \delta\mu$$

Conditioning gradients:

- tapering to suppress large gradient values near source and receiver locations
- tapering to linearly increase the gradient scales with depth to better resolve deeper structures

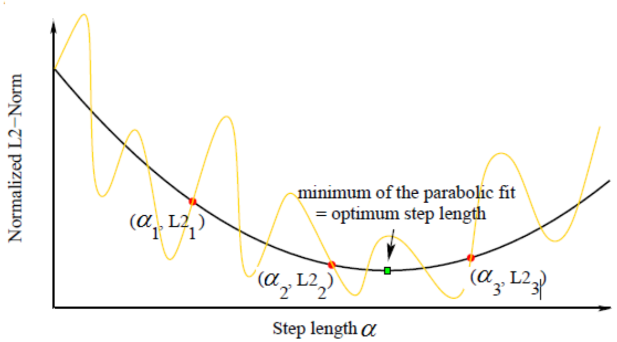
Regularization: $\delta^* V_P = R_{V_P}(LV_P) + \delta V_P$

$$\delta^* V_S = R_{V_S}(LV_S) + \delta V_S$$

Model update:

$$V_P^{\,n+1} = V_P^{\,n} - \alpha_P \delta^* V_P$$

$$V_S^{\,n+1} = V_S^{\,n} - \alpha_S \delta^* V_S$$



3D FWI

➢ Model updating by Gauss-Newton

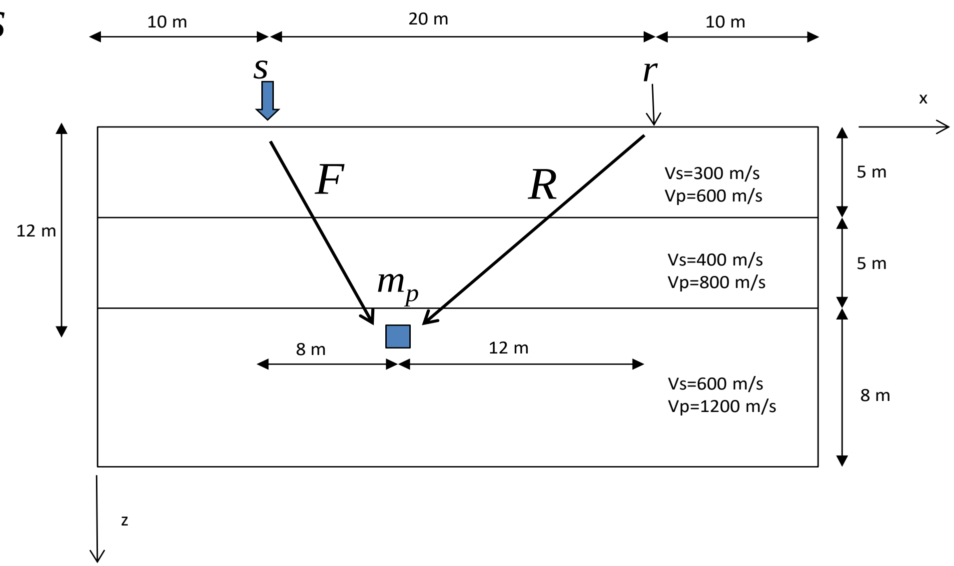
- Velocity residual: $\Delta \mathbf{d}_{i,j} = \mathbf{F}_{i,j}(\mathbf{m}) - \mathbf{d}_{i,j}$
- Misfit function: $\mathrm{E}(\mathbf{m}) = \frac{1}{2} \Delta \mathbf{d}^t \, \Delta \mathbf{d}$
- Model updating: $\mathbf{m}^{n+1} = \mathbf{m}^n - \alpha^n [\overbrace{\mathbf{J}^t \mathbf{J} + \lambda_1 \mathbf{P}^t \mathbf{P} + \lambda_2 \mathbf{I}^t \mathbf{I}}^{\text{Filter, focus, balance gradient vector}}]^{-1} \mathbf{J}^t \Delta \mathbf{d},$
- Jacobian matrix: $\mathbf{J}_{i,j} = \dfrac{\partial \, \mathbf{F}_{i,j}(\mathbf{m})}{\partial \, m_p}$
- Gauss-Newton inversion is done in frequency domain to reduce RAM

$$\tilde{u}(\mathbf{x}, \omega) = \sum_{l=1}^{nt} \exp(\sqrt{-1}\,\omega l \Delta t)\, u(\mathbf{x}, l\Delta t)\Delta t$$

Derivative wave-field (Jacobian Matrix J)

- **Explicit**
  - two forward simulations with and without the model perturbation for each unknown
  - Required number of forward simulations = *number of shots* × (*number of unknowns* +1)

$$J_{i,j}^{p} = \frac{\partial \mathbf{F}_{i,j}(\mathbf{m})}{\partial m_p} = \frac{\mathbf{F}_{i,j}(\mathbf{m}+\Delta m_p) - \mathbf{F}_{i,j}(\mathbf{m})}{\Delta m_p}$$



- **Implicit**
  - Virtual source (F) and reciprocal wave-fields (R)
  - Required number of forward simulations = (*number of shots* + *number of receivers*)

$$J_{i,j}^{p} = F_x * R_x + F_y * R_y + F_z * R_z$$

Virtual source for Vs

Wave equations are differentiated with respect a parameter Vs_n:

$$\rho \frac{\partial \dot{v}_i}{\partial Vs_n} = \frac{\partial \sigma_{ij,j}}{\partial Vs_n}$$

Virtual source

$$\rho \frac{\partial \dot{\sigma}_{ij}}{\partial Vs_n} = \lambda \frac{\partial v_{k,k}}{\partial Vs_n} + 2\mu \frac{\partial v_{i,j}}{\partial Vs_n} - 4\rho Vs \frac{\partial Vs}{\partial Vs_n}(v_{k,k} - v_{j,i}) \quad if \quad i = j$$

$$\rho \frac{\partial \dot{\sigma}_{ij}}{\partial Vs_n} = \mu \left(\frac{\partial v_{i,j}}{\partial Vs_n} + \frac{\partial v_{j,i}}{\partial Vs_n} \right) + 2\rho Vs \frac{\partial Vs}{\partial Vs_n}(v_{i,j} + v_{j,i}) \quad if \quad i \neq j$$

Virtual source for Vp

Wave equations are differentiated with respect a parameter Vp_n:

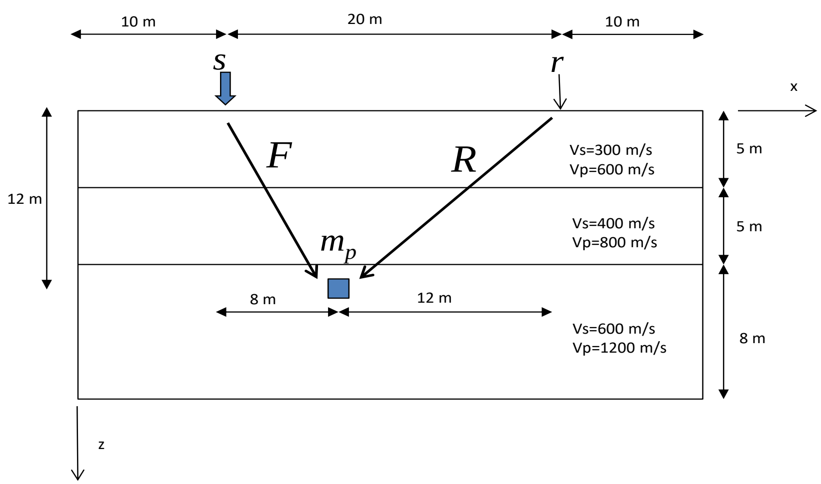
$$\rho \frac{\partial \dot{v}_i}{\partial Vp_n} = \frac{\partial \sigma_{ij,j}}{\partial Vp_n}$$

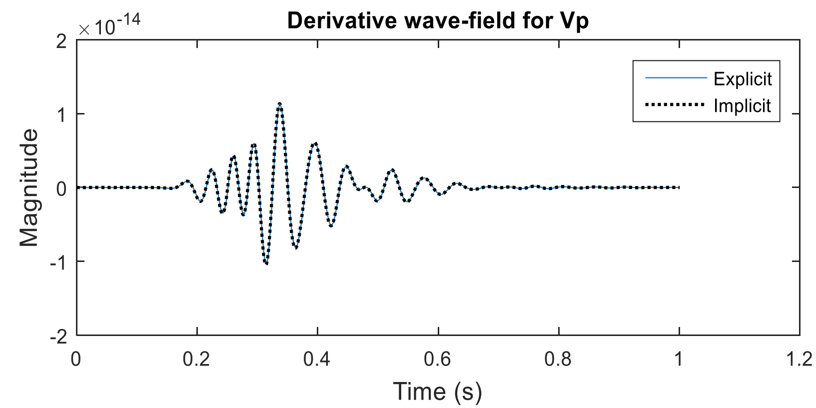
Virtual source

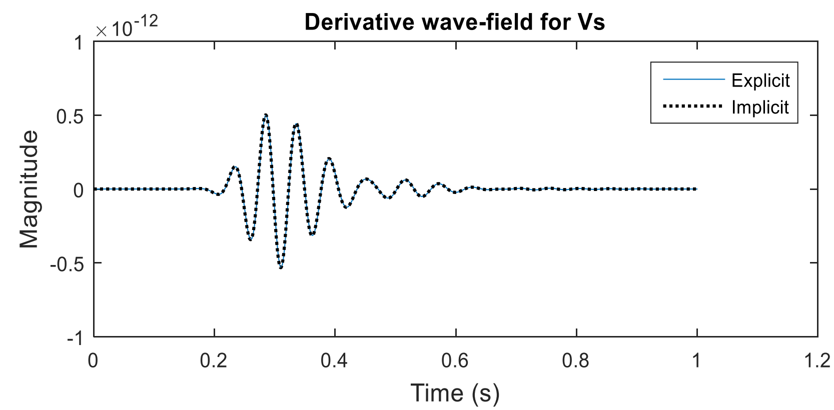
$$\rho \frac{\partial \dot{\sigma}_{ij}}{\partial Vp_n} = \lambda \frac{\partial v_{k,k}}{\partial Vp_n} + 2\mu \frac{\partial v_{i,j}}{\partial Vp_n} + 2\rho Vp \frac{\partial Vp}{\partial Vp_n} v_{k,k} \quad if \quad i = j$$

$$\rho \frac{\partial \dot{\sigma}_{ij}}{\partial Vp_n} = \mu \left(\frac{\partial v_{i,j}}{\partial Vp_n} + \frac{\partial v_{j,i}}{\partial Vp_n} \right) \quad if \ i \neq j$$

Comparison of derivative wave-field



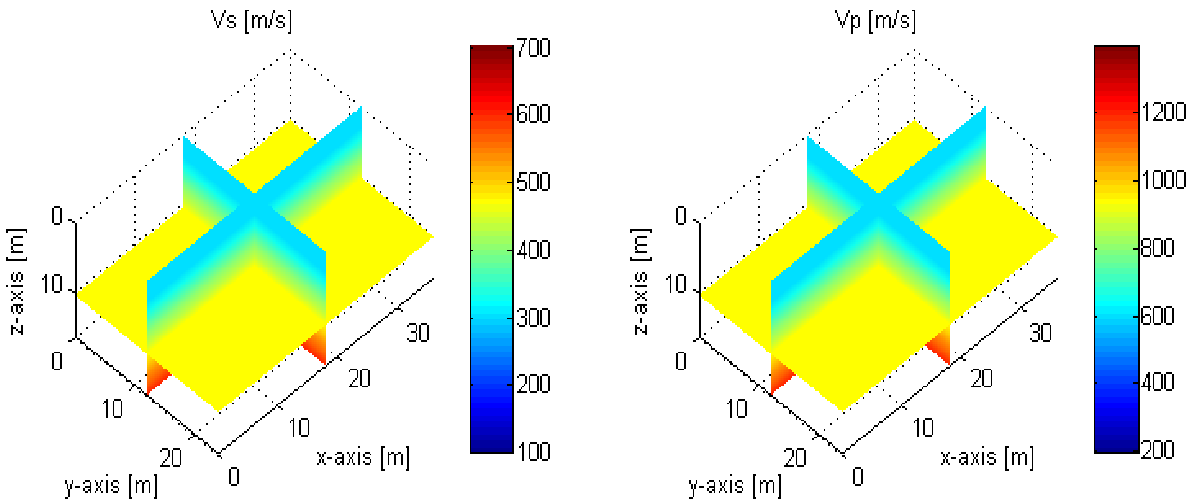




40 x 40 x 18 m
model of 3 layers

Explicit and Implicit are identical

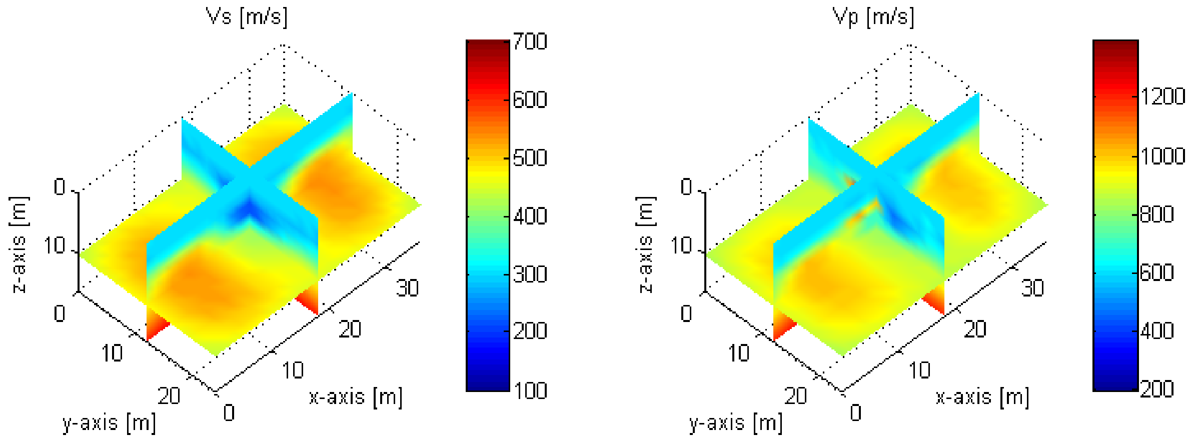
3D FWI: Synthetic test



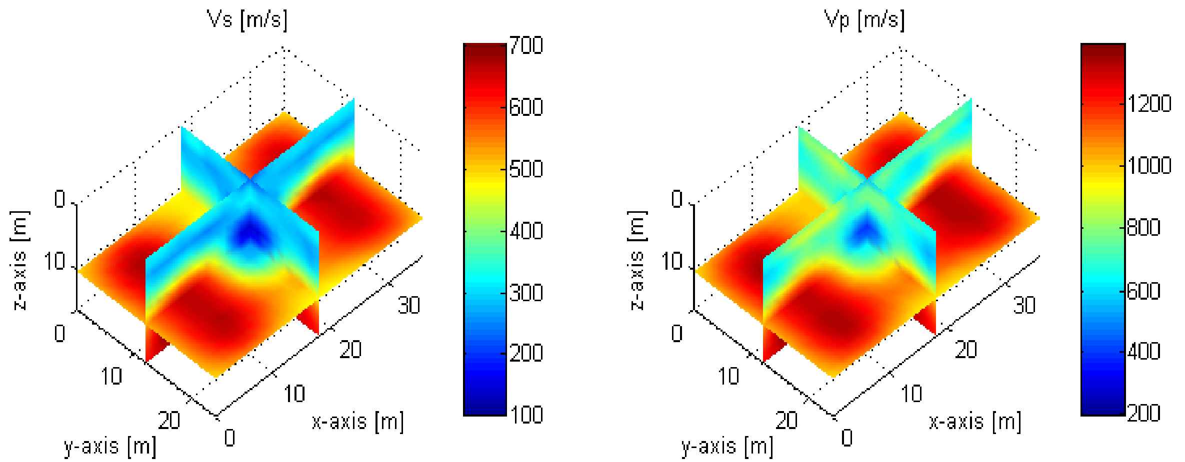
Initial model used for both
Adjoint and GN inversion

- 2 inversion runs at 15 and 25 Hz central frequencies
- about 40 hours for both Adjoint gradient and Gauss-Newton inversions on a desktop computer (32 cores of 3.46 GHz each and 256 GB of memory)

3D FWI: Synthetic test results



Adjoint gradient

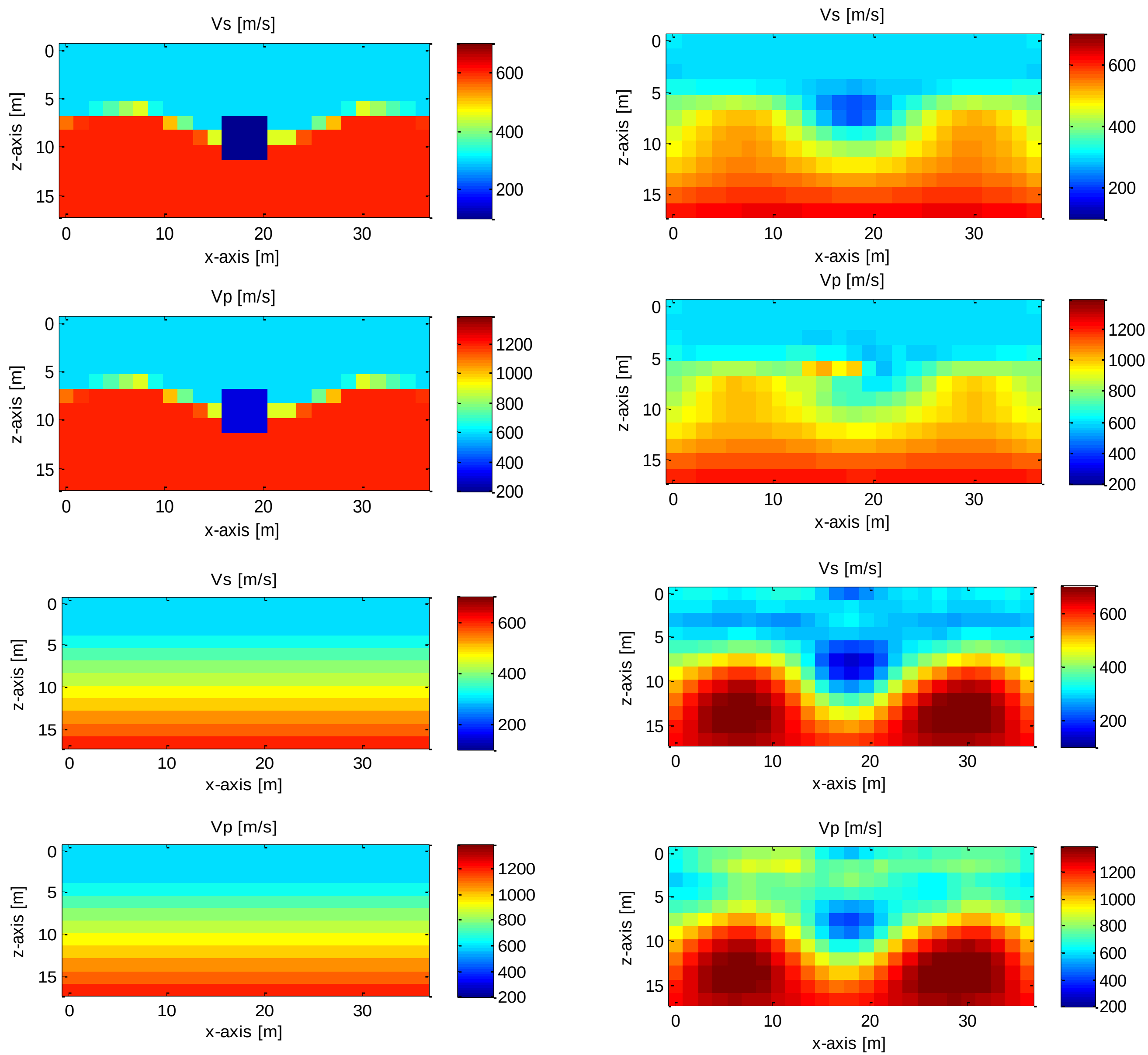


Gauss-Newton

3D FWI: plane comparison at void center

True model

Initial model



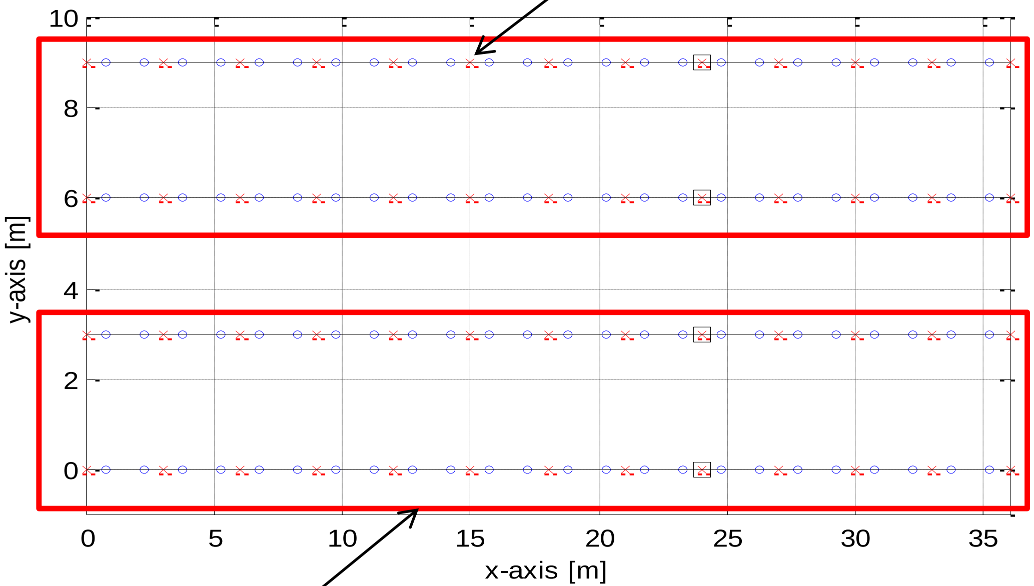
Adjoint gradient

Gauss-Newton

Gainesville site

- dry retention pond in Gainesville, FL
- test area of 36 x 9 m
- 96 receivers located in 24 x 4 grid
- 52 shots located in 13 x 4 grid
- 48 geophones twice
- PEG active source

Stage 1

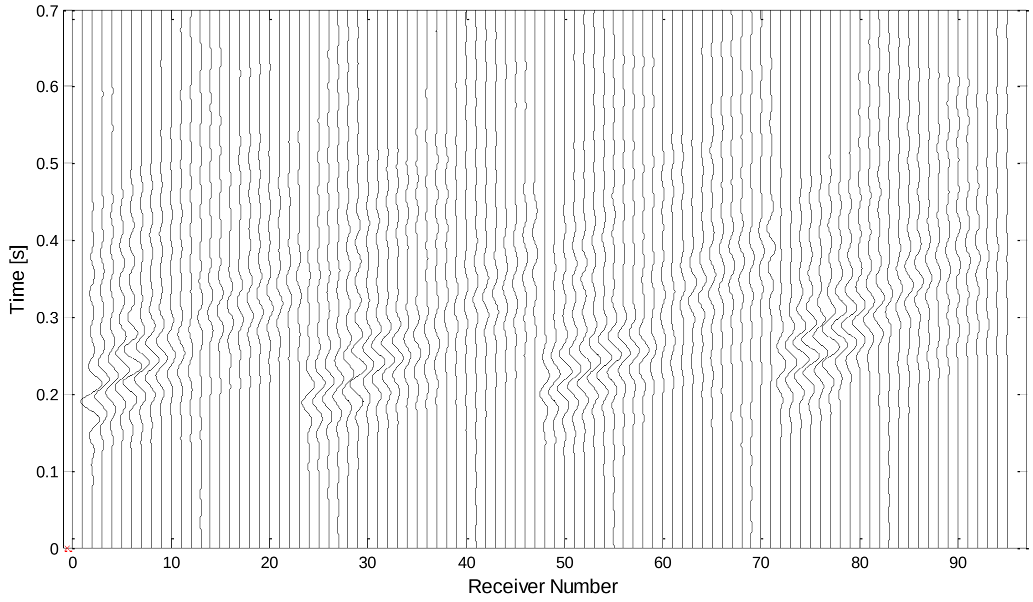


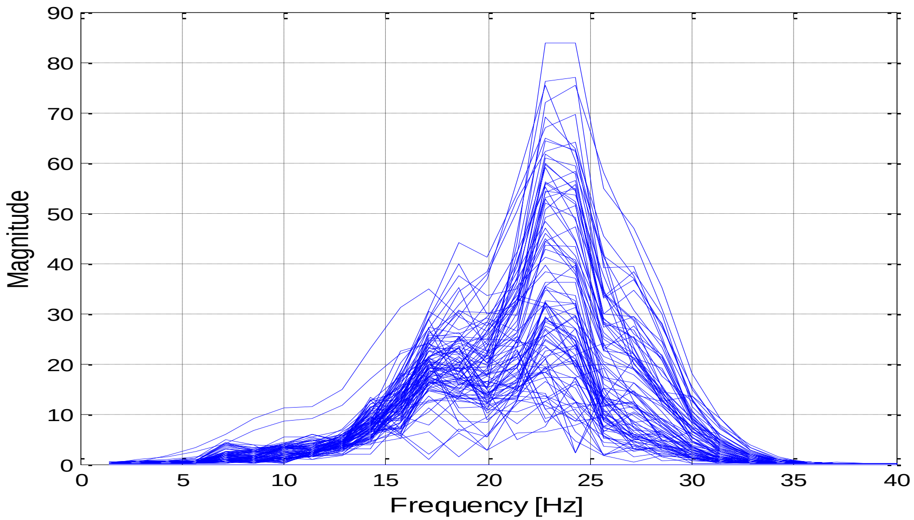
Stage 2

Gainesville site: sample field data

- measured data combined from the two stages for 96-channel shot gather

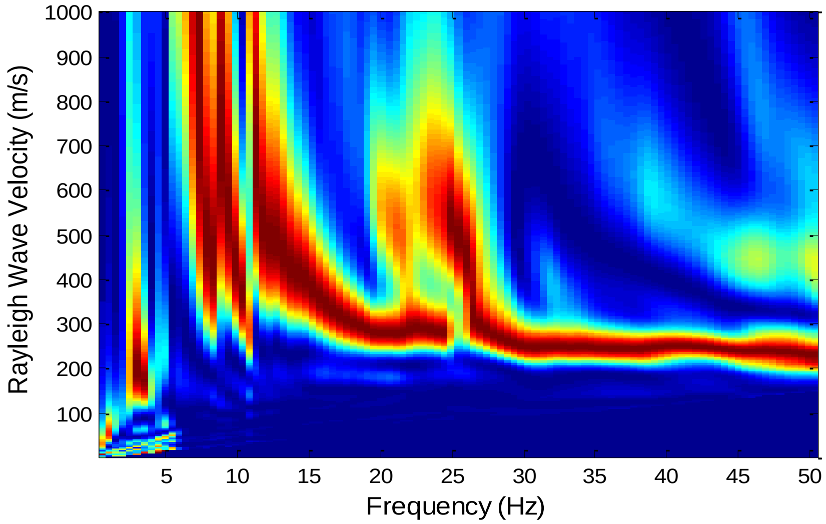
- consistent wave magnitudes and propagation pattern



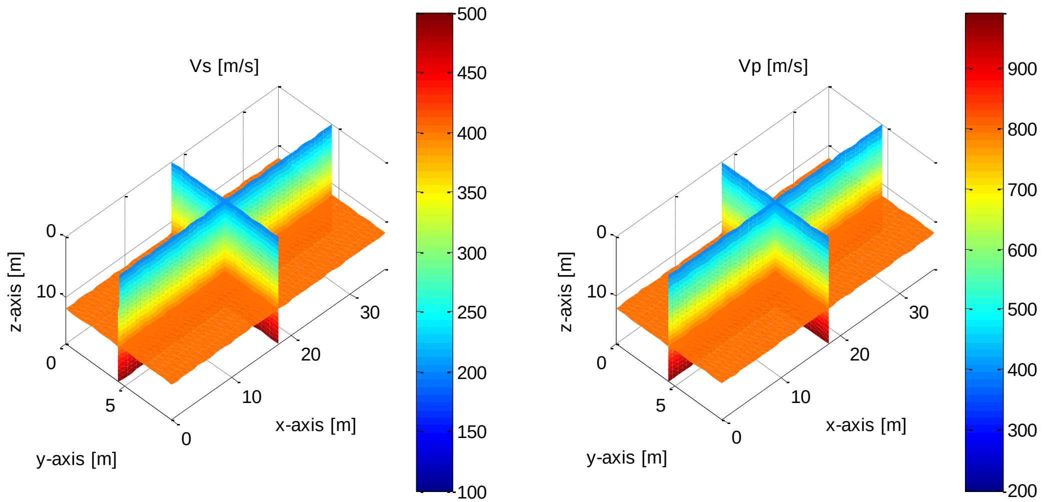


Gainesville site: data analysis

- 2 inversion runs at 12 and 22 Hz central frequencies
- Medium divided into about 14,000 cells of 0.75 m
- About 30 hours for both Adjoint gradient and Gauss-Newton methods

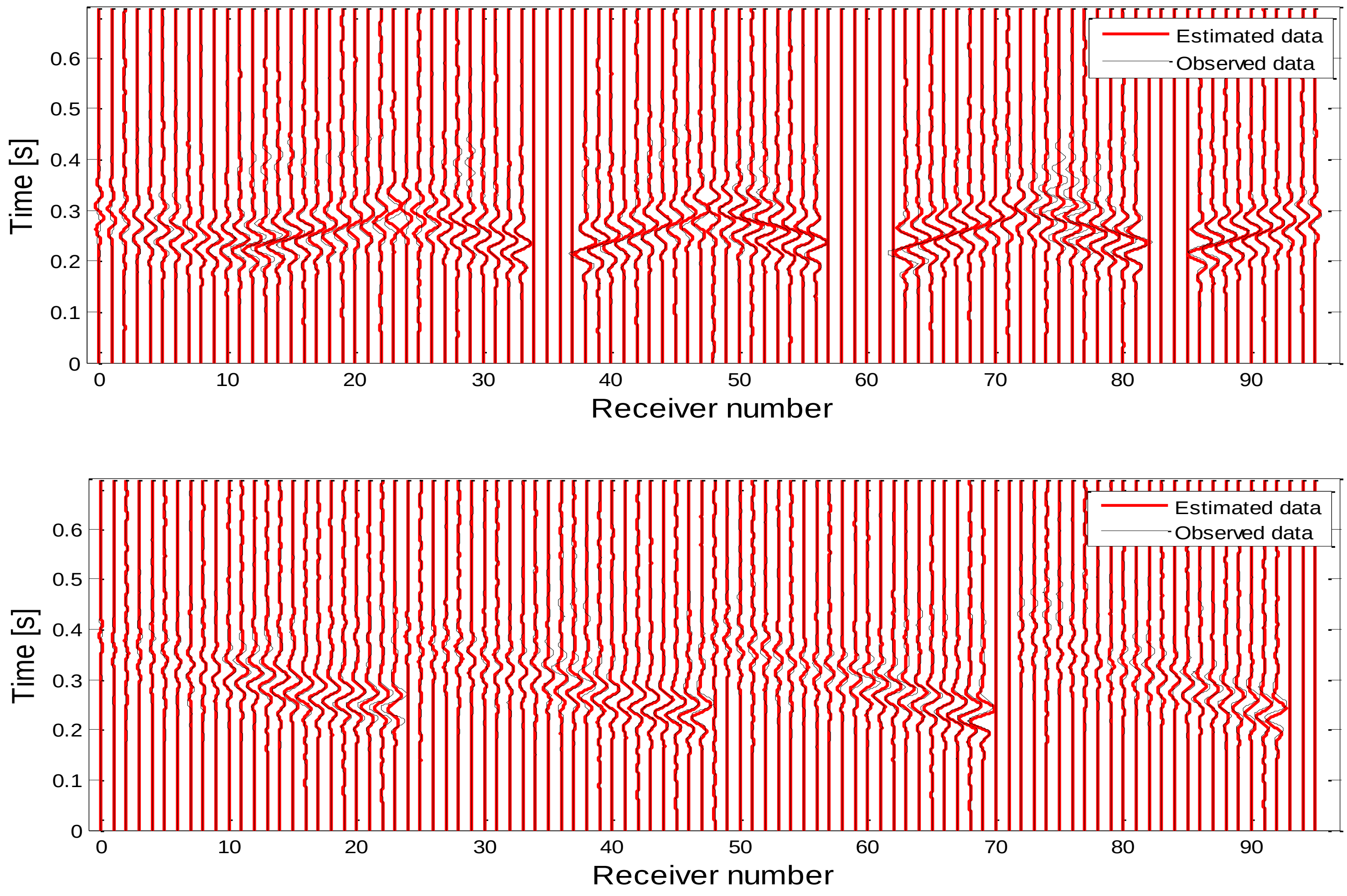


- Power spectrum



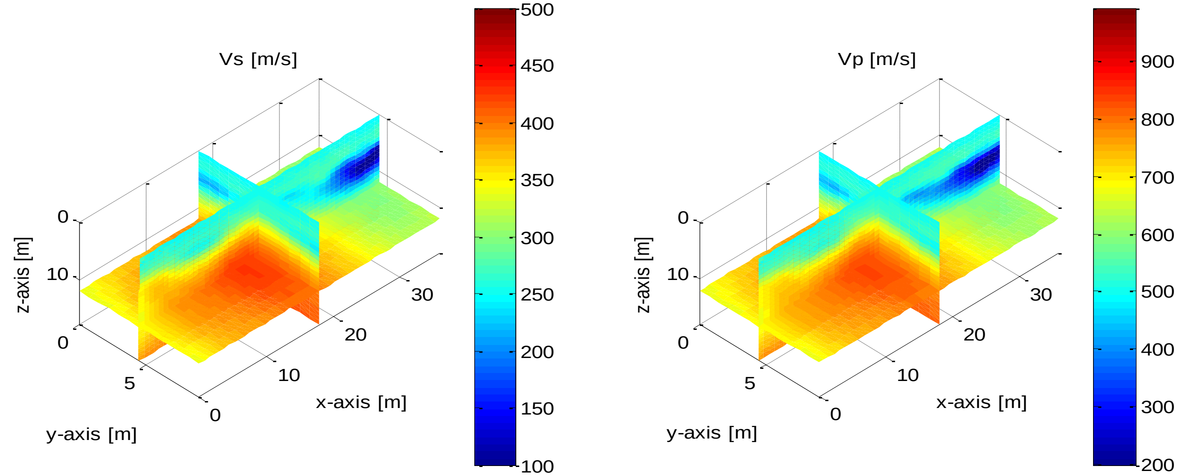
- Initial model

Gainesville site: data analysis

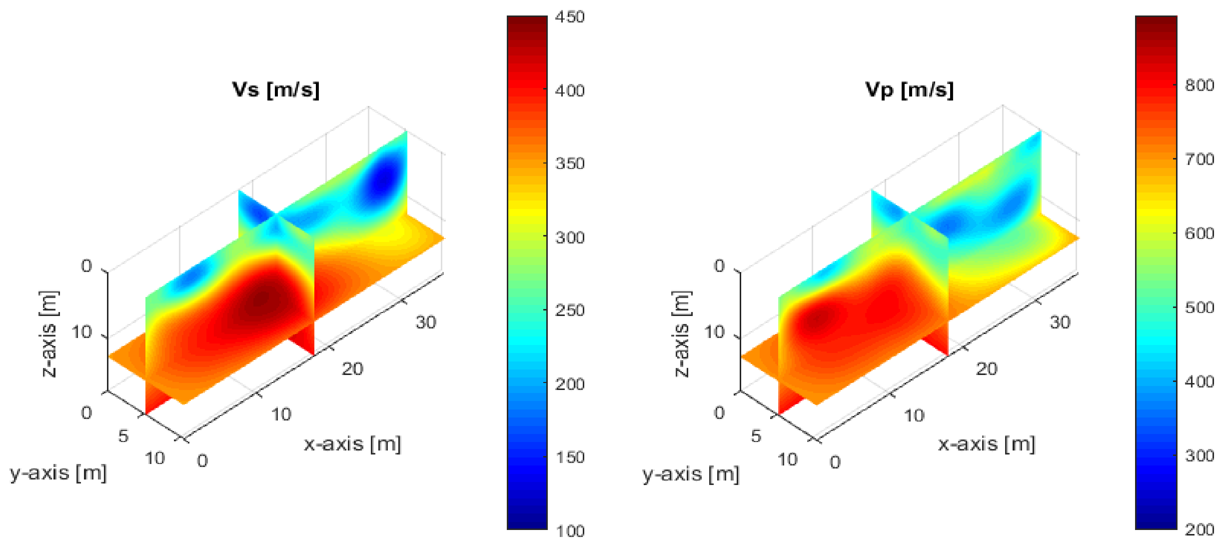


Waveform comparison for 2 sample shots

Gainesville site: 3D FWI results



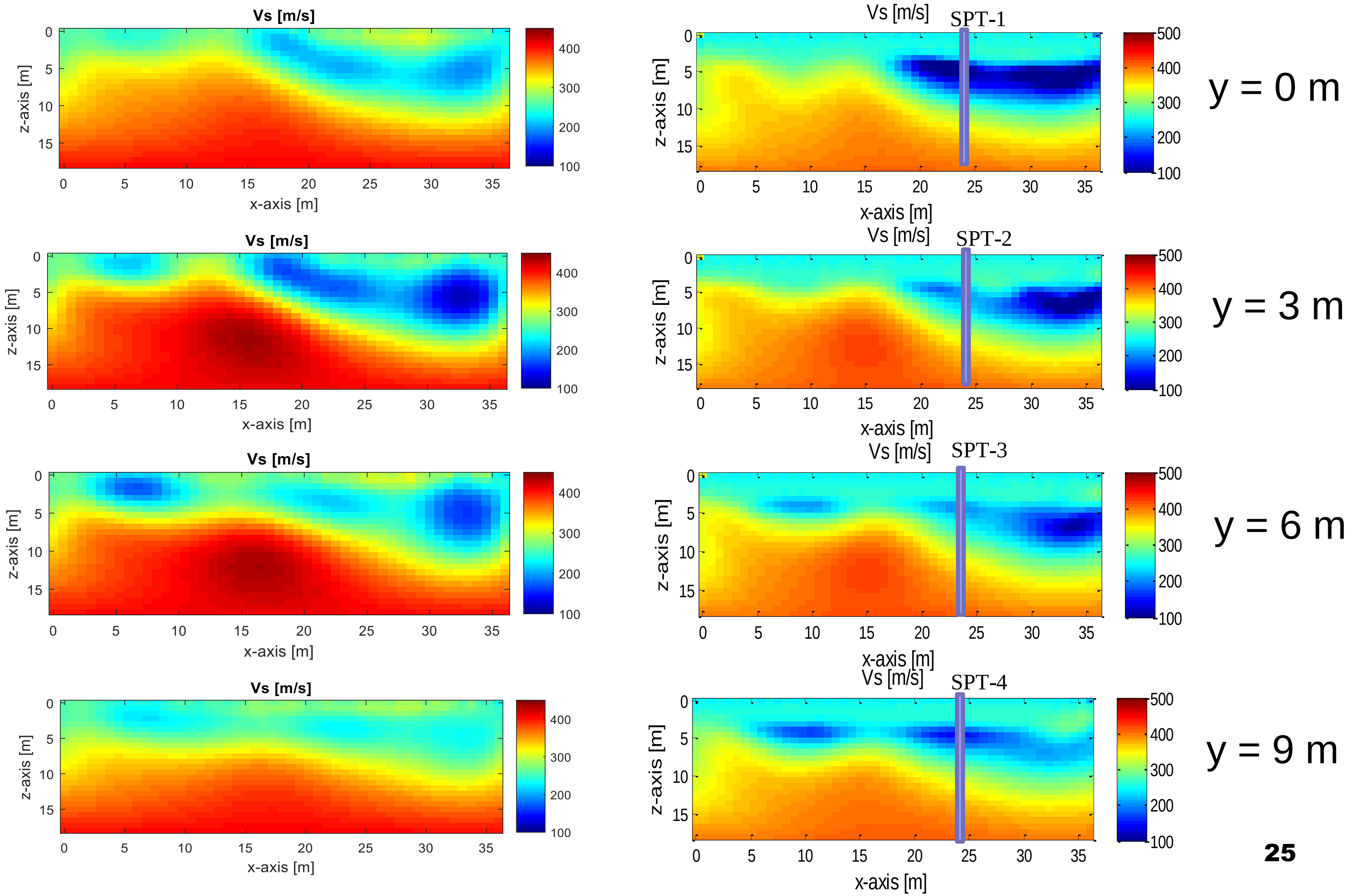
Adjoint gradient



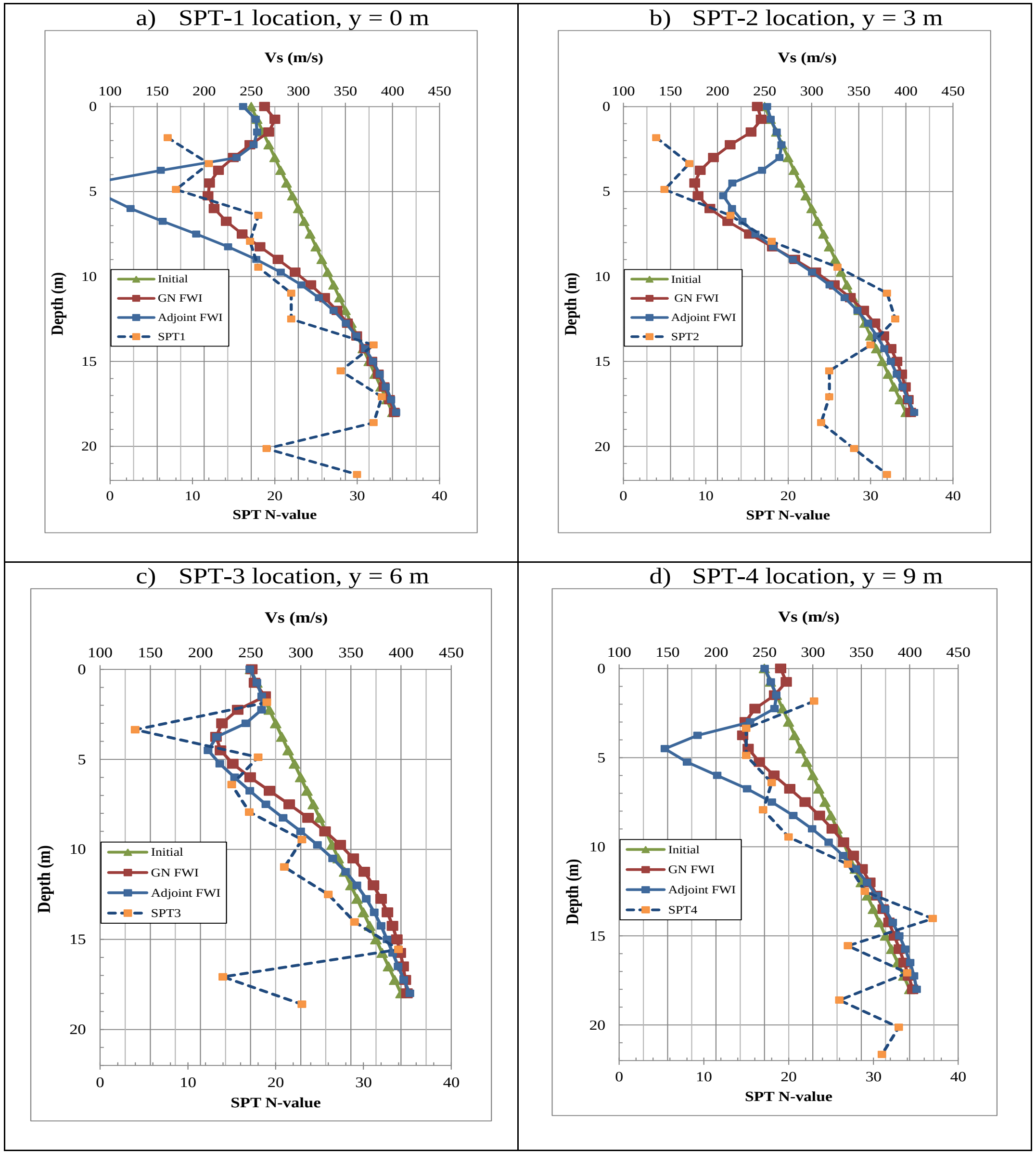
Gauss-Newton

Gainesville site: 3D FWI results at planes

Gauss-Newton | Adjoint Gradient



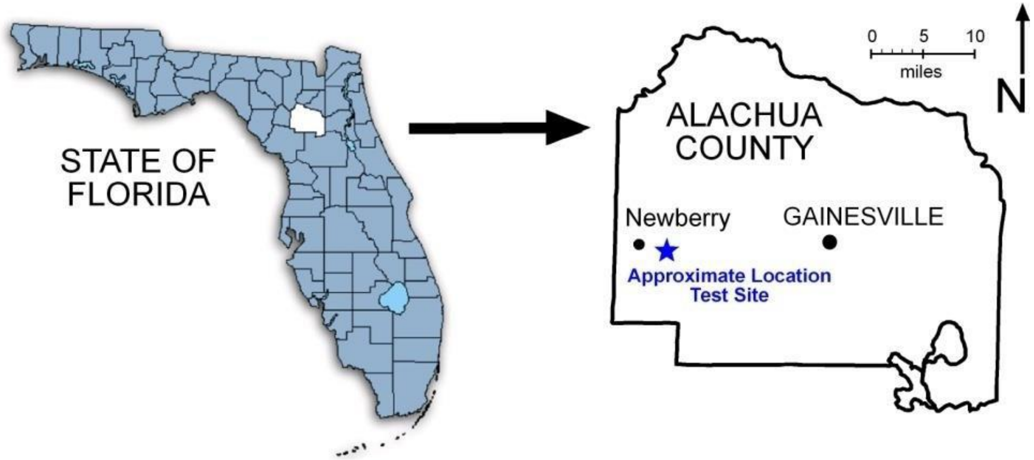
3D FWI vs. SPT results

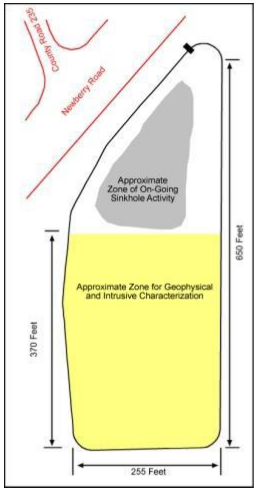


Tran K.T, Mirzanejad M. McVay M. and Horhota D. (2018), "3D Time-Domain Gauss-Newton Full Waveform Inversion for Near-Surface Site Characterization", *Geophysical Journal International*, 2nd review

Newberry site

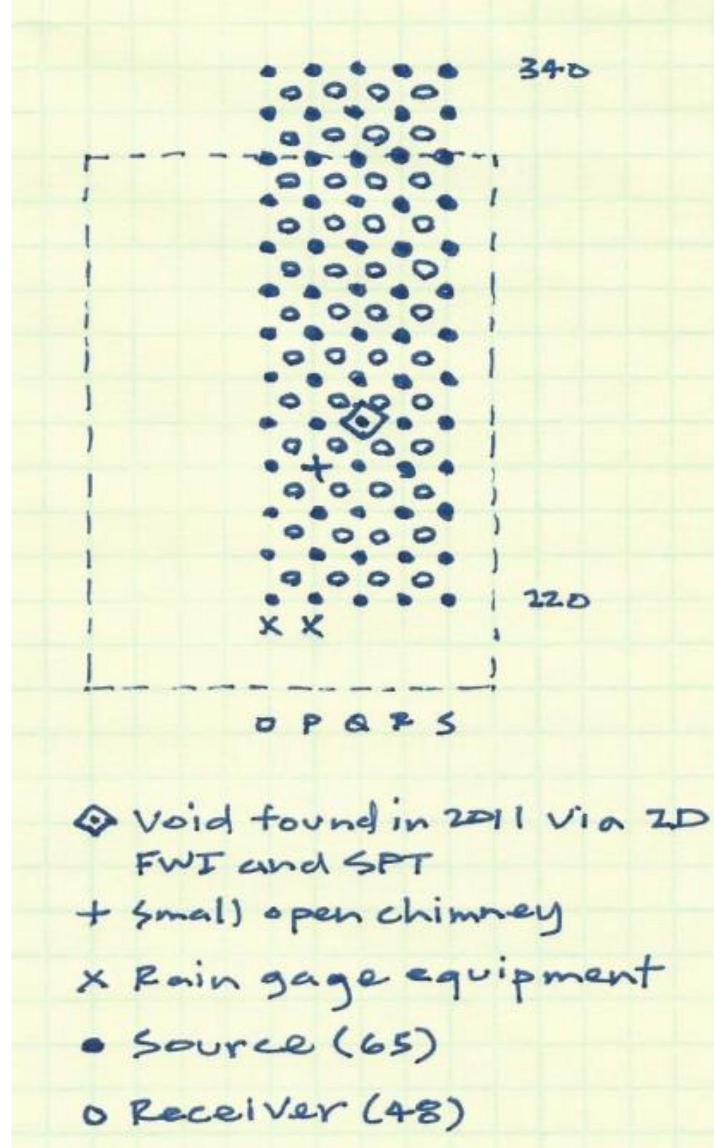
- Dry retention pond in Newbery, FL
- Mix sand and clay over lime stone bedrock
- Site was marked by 25 lines (A to Y) at 3 m spacing
- Data were collected by NHERI @UTexas team using 48 4.5 Hz vertical geophones and Thumper shaking source.





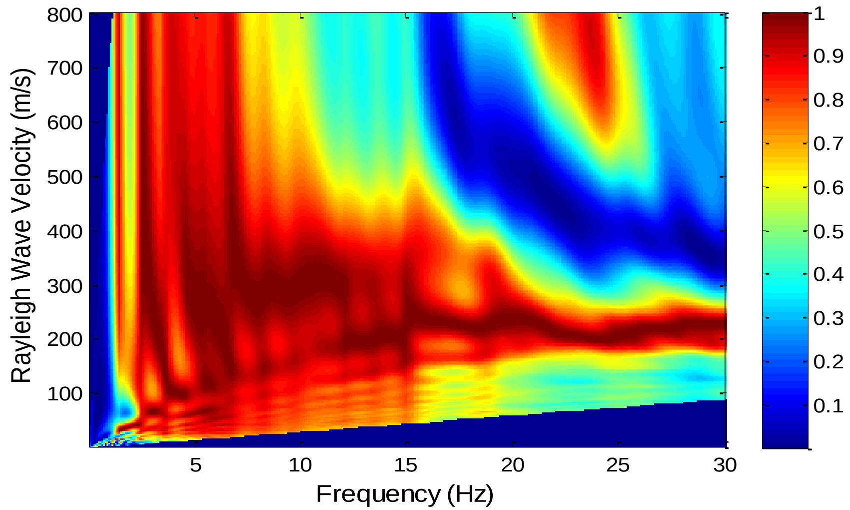
Seismic Survey at Lines O to S

- Test area of 36 x 12 m
- 48 geophones located in 12 x 4 grid
- 65 shots located in 13 x 5 grid
- Thumper shaker source

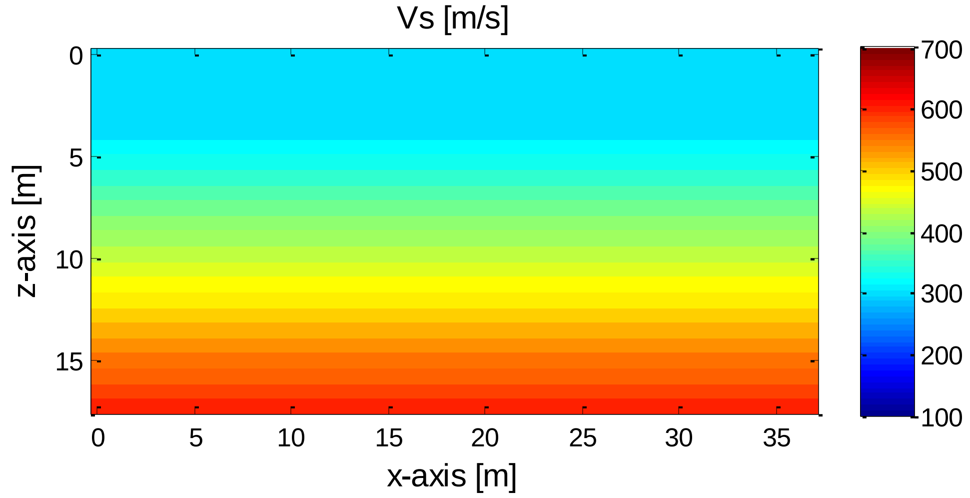


Newberry data analysis

- Analyzed by Gauss-Newton 3D FWI
- 2 inversion runs at 10 and 25 Hz central frequencies
- 60 hours on a desktop computer (32 cores of 3.46 GHz each and 256 GB of memory)



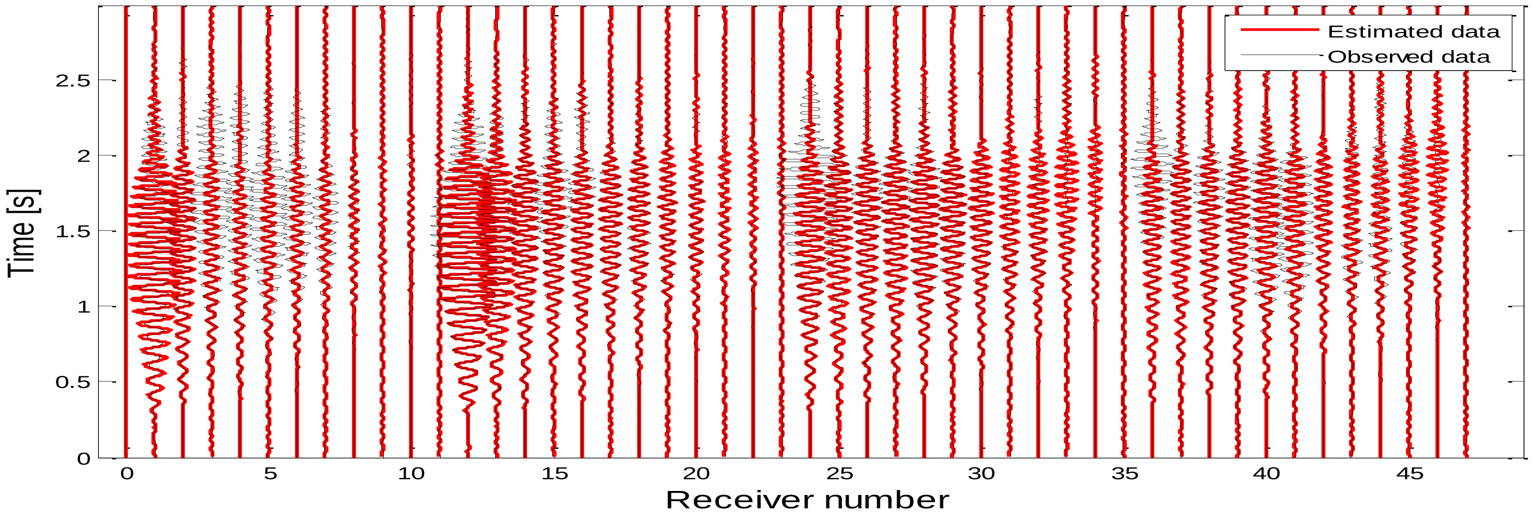
- Power spectrum



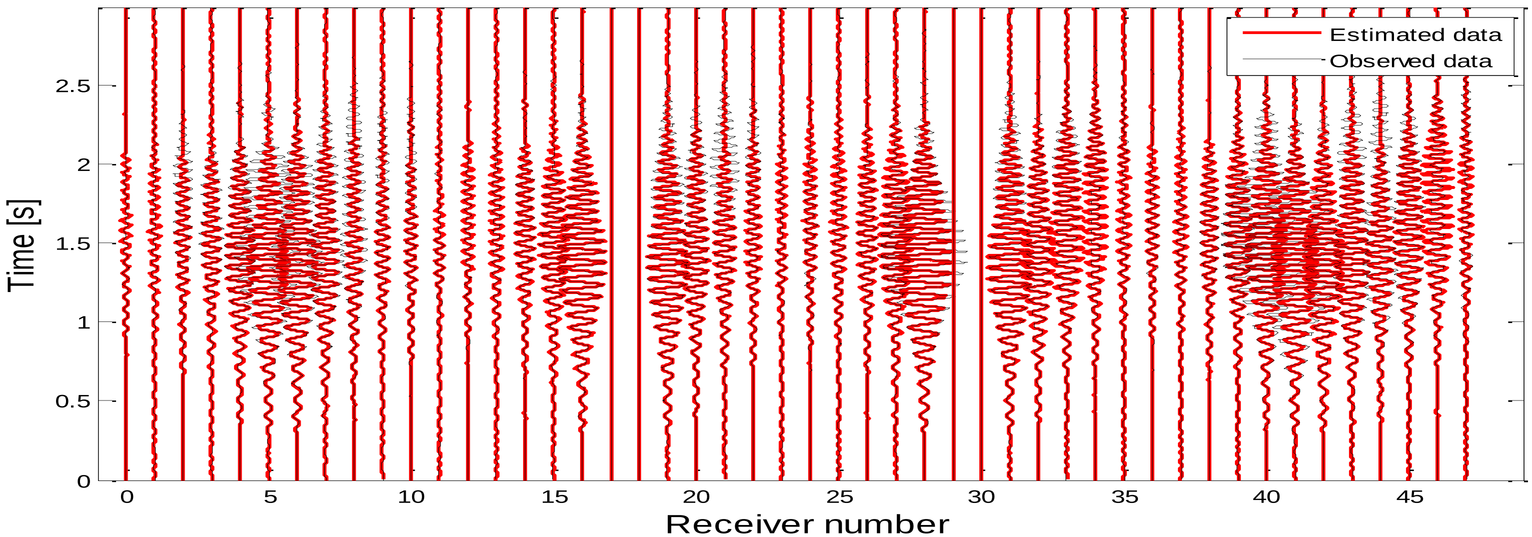
- Initial model

Newberry: data analysis

Shot 1
Line O
x= 0m
y= 0m



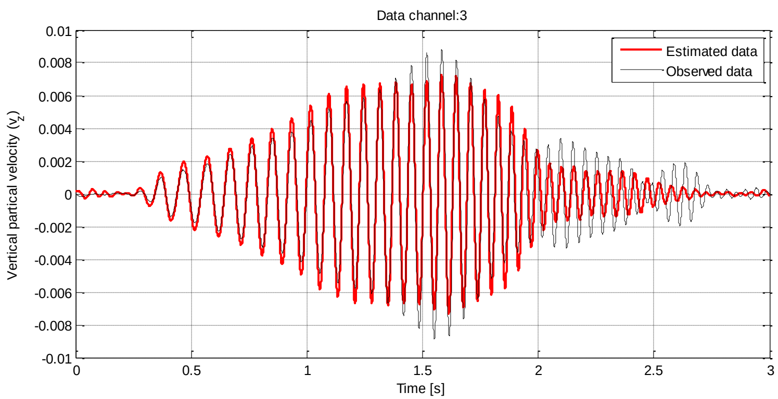
Shot 33
Line Q
x= 18m
y= 6m



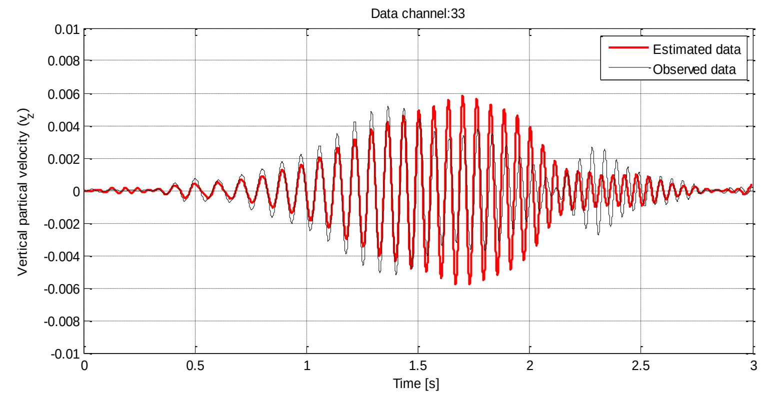
Waveform comparison for 2 sample shots

Newberry: data analysis

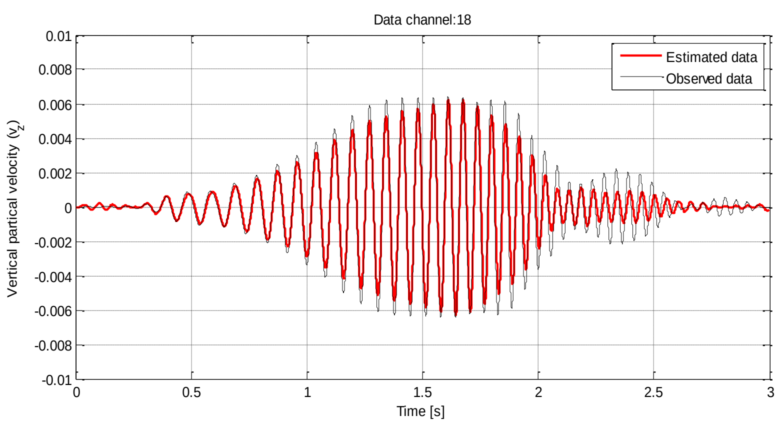
Channel 3, Line OP, x= 7.5m, y= 1.5m



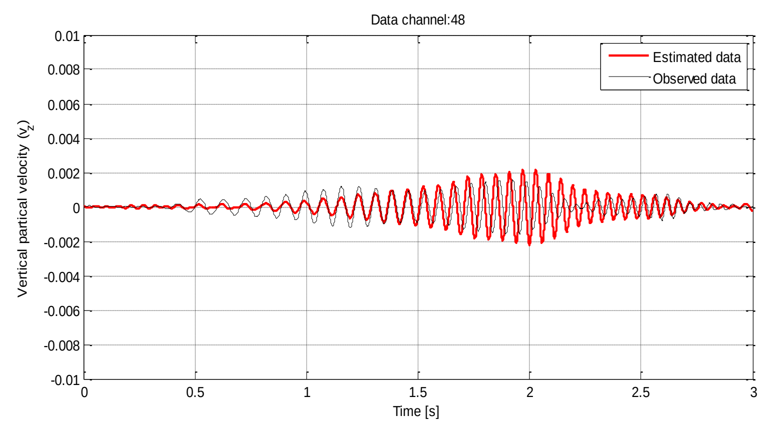
Channel 33, Line QR, x= 25.5m, y= 7.5m



Channel 18, Line PQ, x= 16.5m, y= 4.5m

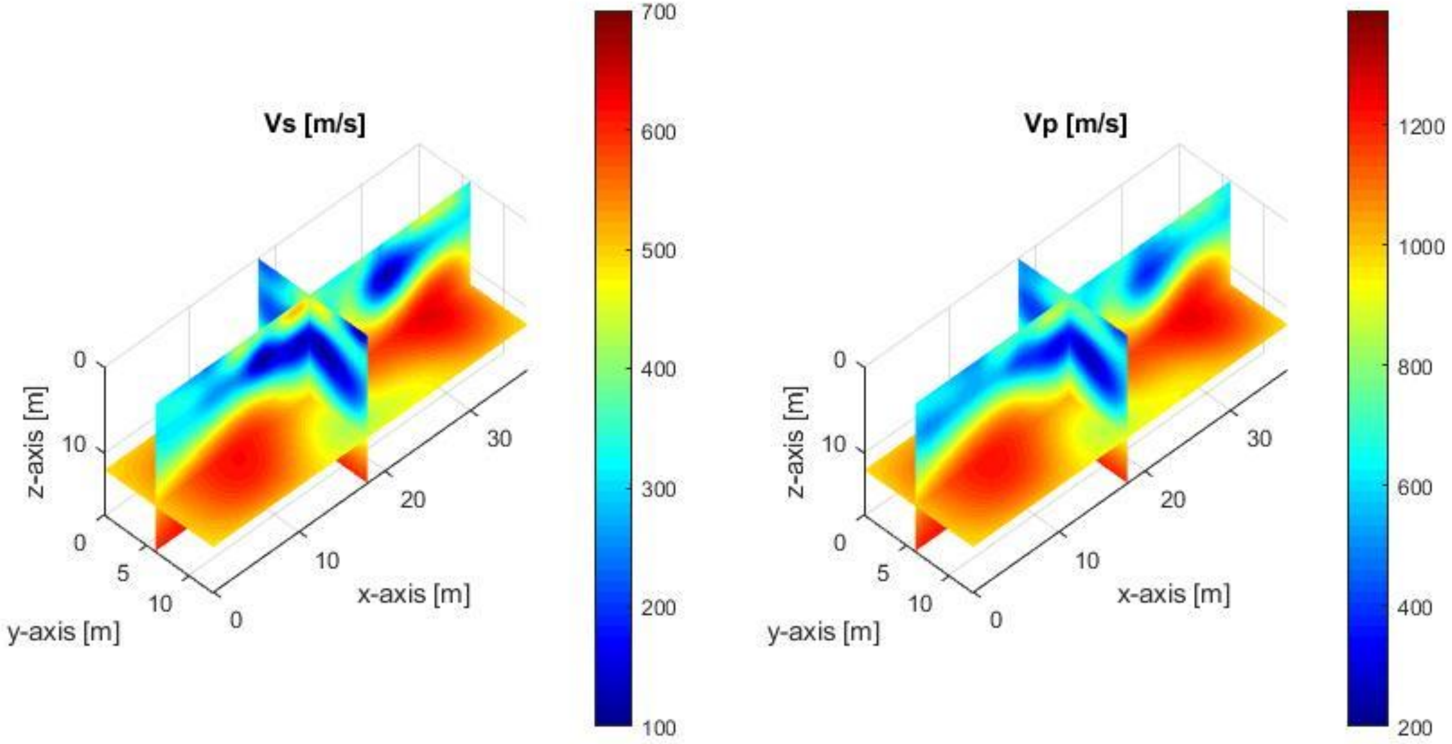


Channel 48, Line RS, x= 34.5m, y= 10.5m



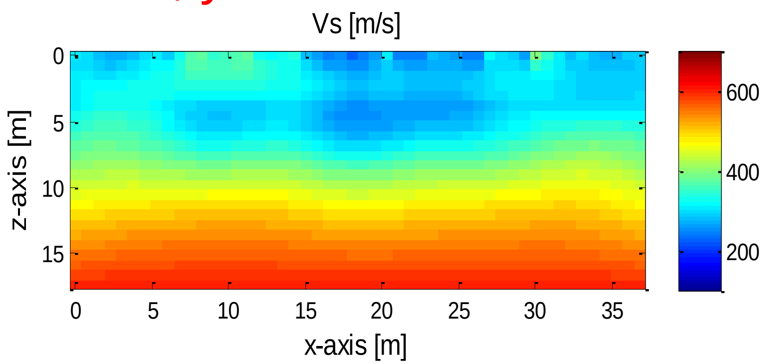
Waveform comparison for 4 sample channels for shot 1 at x=0m, y=0m

Newberry: 3D FWI Results

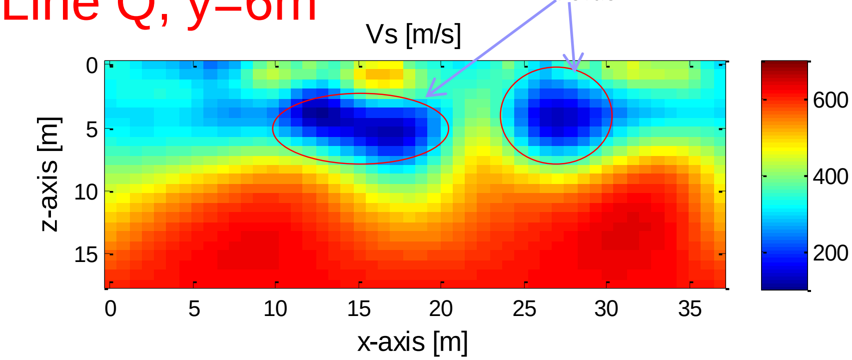


Newbery: 3D FWI Results at planes

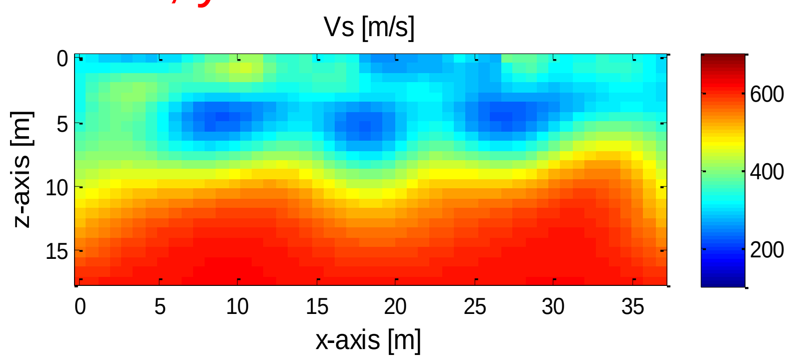
Line O, y=0m



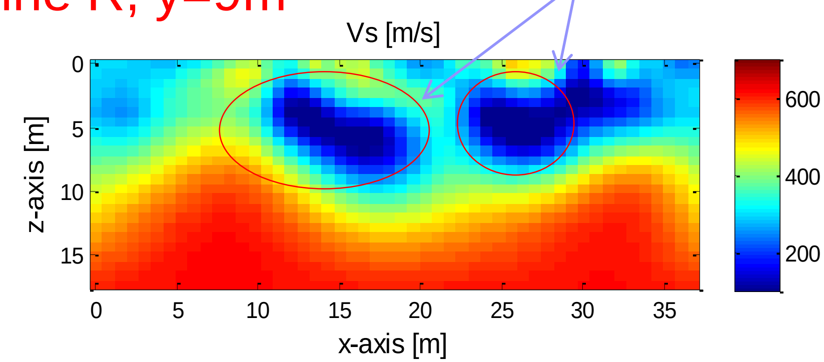
Line Q, y=6m



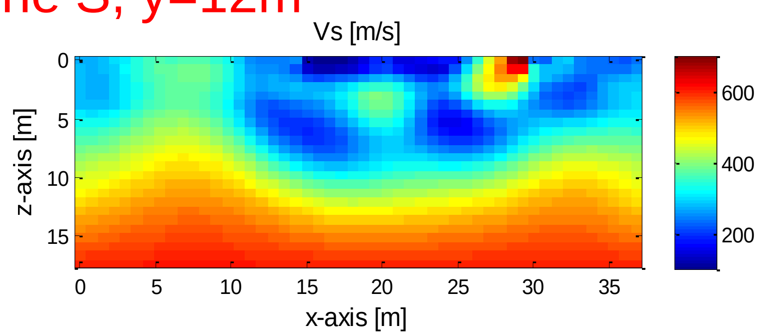
Line P, y=3m



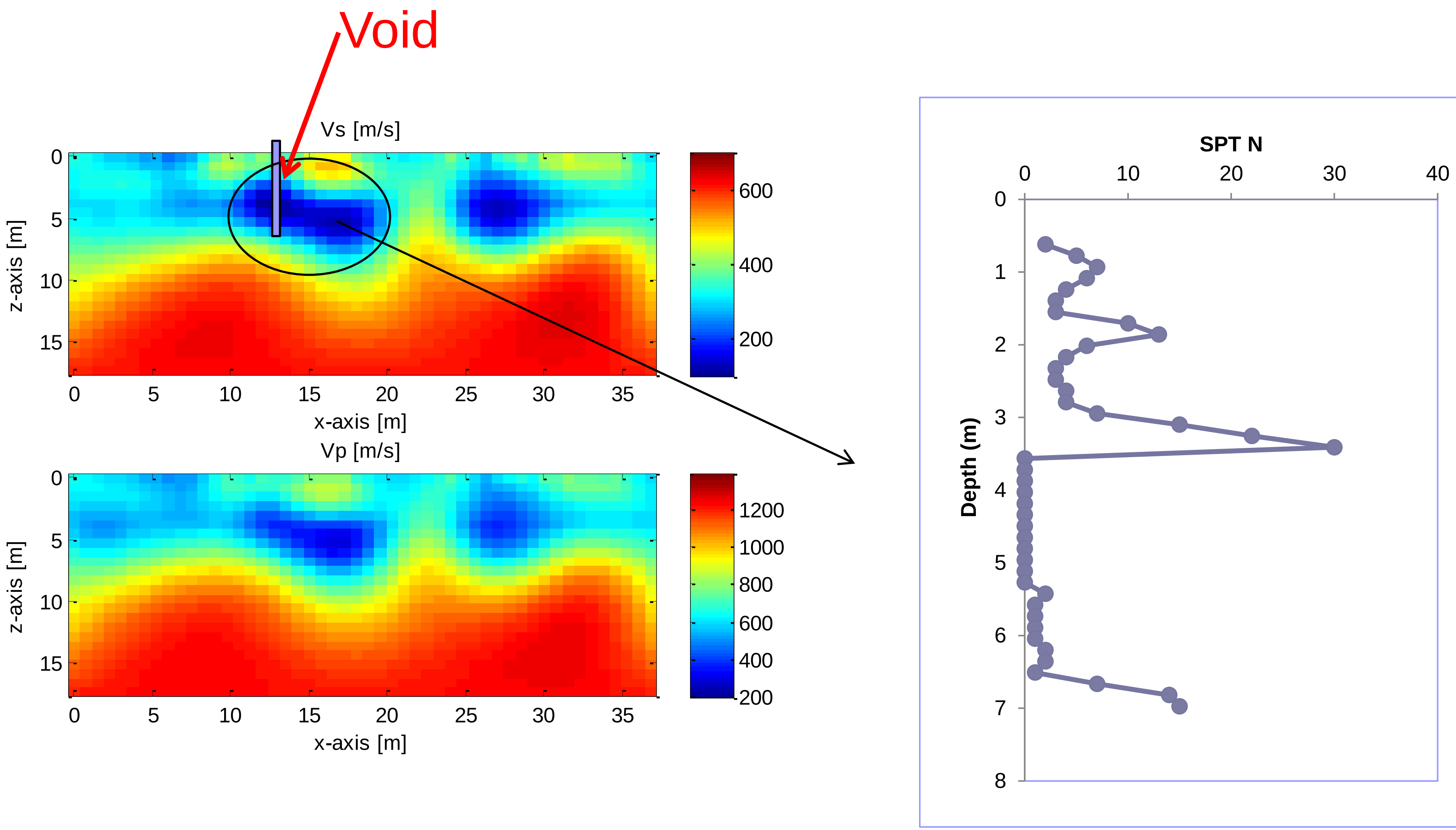
Line R, y=9m



Line S, y=12m



Newbery: Void at Line Q



Conclusion

- Both Vs and Vp can be characterized at high resolution (meter pixel) to 20 m in depth by 3-D FWI methods
- Buried void can be identified to a depth of about 3 void diameters with surface measurement
- Gauss-Newton provides better results than Adjoint gradient inversion, particularly for void imaging

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- Graduate students at Clarkson and UF: Trung Nguyen, Brian Sullivan, Majid Mirzanejad, Amila Ambegedara

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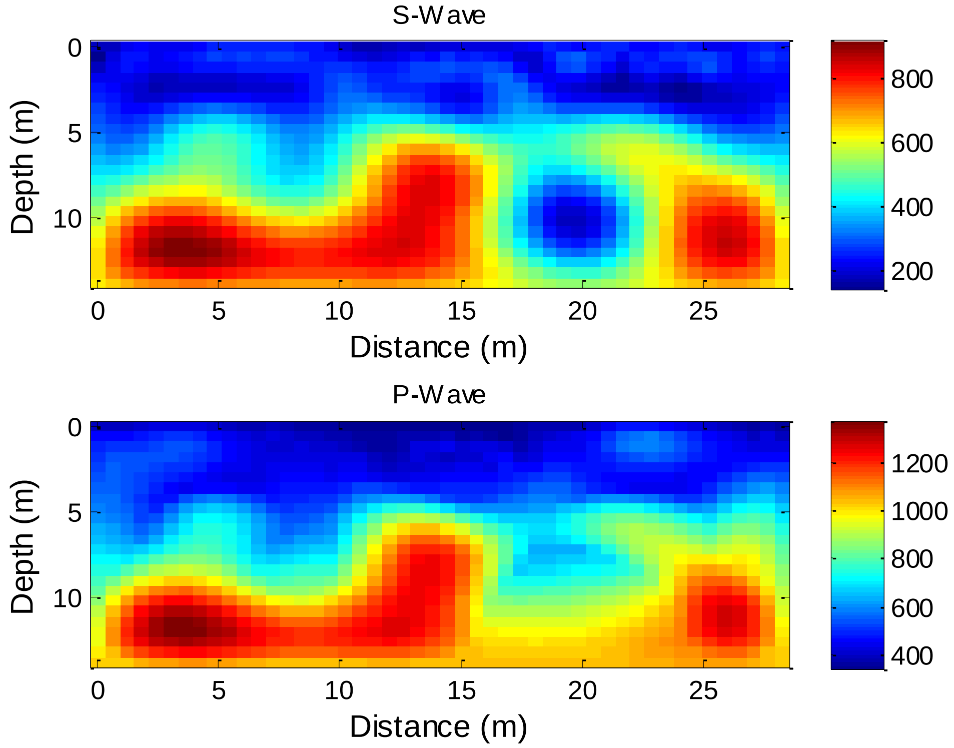
Thank You!


S-Wave
0
5
10
Depth (m)
0
5
10
15
20
25
Distance (m)
800
600
400
200
P-Wave
0
5
10
Depth (m)
0
5
10
15
20
25
Distance (m)
1200
1000
800
600
400